Simulation of the fluorescence detector of the Pierre Auger Observatory

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Abstract

We present a description of a simulation program for the fluorescence detector (FD) of the Pierre Auger Observatory. The simulation chain covers in detail all the physical processes involved in the fluorescence technique, from the shower longitudinal profile in the atmosphere to ADC-traces in the data acquisition system of the telescopes. Steps in the simulation include the generation of fluorescence and Cherenkov light in the atmosphere, propagation of this light to the telescope aperture, ray-tracing of photons in the Schmidt optics of the telescopes, and finally, simulation of the response of the electronics and the multi-level trigger. As an example of the simulation’s use we show the results of a calculation of the trigger efficiency of the FD as a function of cosmic ray energy.

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1. Introduction

The Pierre Auger Observatory is being constructed by an international collaboration, with the aim of studying, with good statistics, the origin of the highest energy cosmic rays ($E > 10^{19}$ eV). It
is a hybrid experiment consisting of an array of surface detectors and a set of air fluorescence telescopes situated on the border of the array. Prototypes of both detectors have been successfully operated and tested at the experiment site, and cosmic ray data is being collected as the expansion of the array proceeds. The general design and status of the experiment has been recently described in [1,2].

Here we present a description of the software used in the Auger Collaboration to perform fluorescence detector (FD) simulations. The present code, named FDSim [3], was developed completely in C++ during the prototype phase of the Observatory. The project is based upon previous Fortran routines which were used in FD design studies in the early phase of the experiment. The aim of the FDSim software is to accurately reproduce the behavior of the FD by means of computer simulations. The role of such a simulation package is twofold. First, it is an extremely useful tool for investigating the response and efficiency of the FD under different configurations and operating conditions, and can be used to optimize the design and evaluate the detector aperture. Secondly, it can be used in testing and optimizing software for the reconstruction of real events. Because of its importance, the code has been constantly tested and improved, and it has now reached a detailed description of the detector features and the physics involved.

In the following sections, we discuss the simulation scheme and the physics modeling used throughout the code. We demonstrate the use of the simulation in Section 7 by studying the trigger rate of the FD for different energies under different atmospheric conditions. Conclusions and an outlook are presented in Section 8.

2. Simulation and modeling

The starting point of the simulation is the creation of objects to describe the atmosphere, the detector, and the air shower. All these objects are instantiated from datacards. The object for the atmosphere contains its description, profiles of pressure and temperature with height, and the physical methods used to calculate the generation, attenuation and scattering of light. The detector object contains information about the FD geometry and setup required for the simulation—for example, telescope and pixel pointing directions, mirror aperture size, optical parameters, etc. In the shower object one can find parameters like the shower geometry and the longitudinal profile. The latter can either be generated from a Gaisser–Hillas parameterization [4] or read from an external file. Once these objects are created, the shower is segmented along its longitudinal propagation axis and the fluorescence and Cherenkov light generated in each segment is propagated to the telescopes, taking into account light attenuation and scattering in the atmosphere. A ray-tracing module is used to propagate photons from the entrance diaphragm of a telescope to the photocathodes of photomultiplier tubes (PMTs). An intermediate simulation result is summarized in a signal object, which contains a list of PMTs each with a histogram of received photons as a function of time. This photon signal is used as input in the final part of the simulation and the modeling of the detector electronics and trigger. Once complete, the simulation data are written to a file in the same format as the real data from the FD data acquisition system, and can be analyzed like a real event.

3. Atmosphere simulation

The fluorescence technique uses the Earth’s atmosphere as a detection medium and a calorimeter. It is the medium where the air showers develop, and where the shower particles interact with the air molecules to generate fluorescence light. In FDSim the default model for the description of the temperature and density profile of the atmosphere is the so-called US standard atmosphere [5]. It is based on experimental data and gives a realistic approximation of the average atmosphere. FDSim uses the Linsley’s parametrization of the US standard atmosphere, as described in [6,7]. Special atmospheric models based on balloon measurements in the experiment site [8] are under development, and they will be consid-
ered in the simulation program beyond the prototype phase of the Observatory.

3.1. Light attenuation

The FD is sensitive to nitrogen fluorescence emission in the wavelength band between 300 and 420 nm. In this band, the primary light attenuation mechanisms are Rayleigh (or molecular) scattering and scattering by aerosols.

The Rayleigh attenuation along a path between two points in the atmosphere, represented by their vertical atmospheric depths $X_1$ and $X_2$ (in g/cm$^2$), is given by [9],

$$\mathcal{F}_{\text{Ray}} = \exp \left[ -\left| \frac{X_1 - X_2}{X_R \cos \theta} \right| \left( \frac{\lambda_R}{\lambda} \right)^4 \right]$$  \hspace{1cm} (1)

where $\theta$ is the zenith angle of the light path, $X_R$ is the Rayleigh attenuation length at $\lambda_R = 400$ nm, and $\lambda$ is the photon wavelength in nanometres. The value of $X_R$ is set in a datacard, and has a default value of 2974 g/cm$^2$.

For the aerosol (or Mie) attenuation of light, the software uses a wavelength independent model, described in [9],

$$\mathcal{F}_{\text{Mie}} = \exp \left[ -\frac{h_M}{\left( \frac{h_1}{h_M} \right)^{1/2} \exp \left( -\frac{h_2}{h_M} \right) \left( \frac{h_1}{h_M} \right)^{1/2} \exp \left( -\frac{h_2}{h_M} \right) \right] \right]$$ \hspace{1cm} (2)

where $l_M$ is the aerosol horizontal attenuation length at sea-level, $h_M$ is the aerosol scale height, $h_1$ and $h_2$ are the altitudes above sea level of the emission and observation points, respectively, and $\theta$ is the zenith angle of the light path. $l_M$ and $h_M$ are also set in a datacard. Typical values are 15,000 and 1200 m, respectively.

Atmospheric measurements from lidars and the Central Laser Facility will be used in the production phase of the experiment in simulation and data analysis. The simple exponential aerosol attenuation described above will be supplanted in FDSim by a layered model derived from those measurements.

The total attenuation is taken as the product of the Rayleigh and Mie contributions:

$$\mathcal{F} = \mathcal{F}_{\text{Ray}} \times \mathcal{F}_{\text{Mie}}.$$  \hspace{1cm} (3)

3.2. Fluorescence light

The fluorescence yield is assumed to be proportional to the energy deposition of the shower in the atmosphere. There are options in FDSim to run simulations either using energy deposition profiles from CORSIKA simulated showers [7,10] or to calculate these quantities from longitudinal charge profiles and using a parametric model. The total fluorescence yield, $F(x)$, at a shower point $x$, is given in units of photons/m by

$$F(x) = \frac{(dE/dX)(x)}{(dE/dX)_{1.4 \text{ MeV}}} \times Y(\rho, T) \quad \text{ (4)}$$

where $dE/dX$ is the energy deposited per g/cm$^2$ at the shower point $x$, $(dE/dX)_{1.4 \text{ MeV}} = 1.676 \text{ MeV/(g/cm}^2\text{)}$ is the energy loss evaluated for an electron of energy 1.4 MeV at standard temperature and pressure [11]; $\rho$ is the atmosphere density in g/cm$^3$, and $T$ is the atmosphere temperature in Kelvin at the emission point, and $Y(\rho, T)$ is the experimental scintillation yield (photons/m) for a 1.4 MeV electron, that being the beam energy available to Kakimoto et al. [12].

The empirical formula for the scintillation yield, $Y(\rho, T)$, is given as in [12] by

$$Y(\rho, T) = \sum_{i=0}^{19} \left( \frac{w(\lambda_i)}{1 - w(391.4 \text{ nm})} \frac{\rho A_1}{F(1 + \rho B_1 \sqrt{T})} \right) + \frac{\rho A_2}{F(1 + \rho B_2 \sqrt{T})} \quad \text{ (5)}$$

where $\lambda_i$ is the wavelength in the center of the $i$th wavelength bin, and $w(\lambda)$ are the weights for the color of the light, taken from the normalized fluorescence spectrum. The constants $F$, $A_1$, $A_2$, $B_1$, $B_2$ are experimental parameters of the fluorescence yield with values $1.044 \times 10^{-5}$, 0.929, 0.574, 1.850, and 6.500, respectively.

Fluorescence photons are emitted isotropically and with a wavelength distribution according to the fluorescence spectrum [12,13].
Bunner [13] estimated with an uncertainty of about 30% the sea-level efficiencies of the fluorescence bands in air. He quoted 22 bands between 300 and 420 nm. Kakimoto et al. [12] did more accurate measurements of the yield for 1.4 MeV electrons at sea-level and 15°C in the primary bands 337, 357 and 391 nm. The obtained yield values of the corresponding bands are 1.109, 1.190 and 0.495 photons/m. A broad-band filter was used additionally to measure the total yield between 300 and 400 nm. They obtained the total value of 3.25 photons/m. The yield of the three primary bands subtracted from the total yield is assumed to be the remaining yield due to the other bands in the interval. The combination of systematic and statistical errors in the experiment was about 10% and 3%, respectively.

In FDSim we use a combination of the results from Kakimoto et al. and Bunner. We use the yield from Kakimoto et al. [12] for the three primary lines, and we distribute the remaining yield from Kakimoto et al. of 0.46 photons/m, between 300 and 420 nm according to Bunner’s efficiency. Twenty bins of wavelength of equal size are used to represent the spectrum from 300 to 420 nm, as shown in Fig. 1.

Geometrical and attenuation factors are used to calculate the number of photons that reach the diaphragm of a telescope in a given small wavelength range. Since photons are isotropically emitted, the geometrical factor, $g$, is given by

$$g(x, x_{eye}) = \frac{A \cos \gamma}{4\pi|\vec{x} - \vec{x}_{eye}|^2}$$

(6)

where $\vec{x}$ is the emission position on the shower axis, $x_{eye}$ is the eye position, $A$ is the area of the diaphragm, and $\gamma$ is the angle between the normal of the diaphragm and the arrival direction of the light.

The light attenuation factor is given by Eq. (3), so the number of photons that reach the diaphragm of a telescope, $N_{Tel}^{ph}$, for a given wavelength bin $\lambda$ is given by

$$N_{Tel}^{ph}(\lambda) = g(x, x_{eye}) \times w(\lambda) \times F \times \Delta \lambda \times \mathcal{F}(x, x_{eye}, \lambda)$$

(7)

where $x$ is the emission position taken to be midway along a track segment of length $\Delta \lambda$, $x_{eye}$ is the eye position, $g$ is the geometrical factor from Eq. (6), $w(\lambda)$ is the weighting factor for the wavelength bin $\lambda$, $F$ is the fluorescence yield as described in Eq. (4), and $\mathcal{F}(x, x_{eye}, \lambda)$ is the attenuation due to the atmosphere as in Eq. (3).

### 3.3. Cherenkov light

Cherenkov light from the shower will contribute to the detected signal. That light may travel in a direct path from the shower to the detector, or it may be scattered in the detector direction by Rayleigh or aerosol scattering. FDSim first calculates the strength of the Cherenkov beam which develops along the shower axis as the shower develops.

The number of Cherenkov photons generated by a particle of energy $E$ in a wavelength bin $\Delta \lambda$ is calculated as in [9]. The formula is given by

$$\frac{dN_{ph}}{dl}(E, \lambda, \Delta \lambda) = 4\pi \delta \left( 1 - \left( \frac{E_{min}}{E} \right)^2 \right) \times \Delta \lambda / \lambda^2$$

(8)

where $dN_{ph}$ is the number of photons generated in the length $dl$ along the shower axis, $\delta = n(h) - 1$ is the difference between the refraction indices $n(h)$ of the air at altitude $h$ and that of the vacuum, $E_{min}$ is the

\[\text{Fig. 1. Fluorescence spectrum assumed in FDSim based on Kakimoto et al. [12]. The arrows indicate the primary bands at 337, 357 and 391 nm. The normalization shows the yield for a 1.4 MeV electron at sea level and 15°C.}\]
Cherenkov energy threshold, \( E \) is the particle energy, and \( \lambda \) is the photon wavelength.

The Cherenkov energy threshold is given by

\[
E_{\text{min}} = \frac{0.511}{\sqrt{2\delta}} \text{ MeV.}
\]

(9)

The shower at any point in its development contains particles with a range of energies. FDSim uses a parametrization of the particle energy spectrum from [14] that depends on the age \( s \) of the cascade. The number of Cherenkov photons within a wavelength range \( \Delta \lambda \) produced in a length of track \( \Delta l \) is

\[
\frac{\Delta N_{\text{ph}}(\Delta \lambda)}{\Delta l} = \Delta l \left[ \sum_{E_i = E_{\text{min}}}^{E_{\text{max}}} \left( F(E_i, s) N_{\text{ch}}(x) \frac{dN_{\text{ph}}}{dl}(E_i, \Delta \lambda) \right) + F(E > E_{\text{max}}, s) N_{\text{ch}}(x) \frac{dN_{\text{ph}}}{dl} \times (E > E_{\text{min}}, \Delta \lambda) \right]
\]

(10)

where in the first term on the right-hand side of the equation, \( F(E_i, s) \), is the fraction of shower particles with energies in a bin centered on \( E_i \), \( N_{\text{ch}}(x) \) is the total number of charged particles at point \( x \) and \( E_{\text{max}} \) is the maximum energy used in the numerical calculation. In the second term on the right-hand side of the equation, \( F(E > E_{\text{max}}, s) \) is the fraction of shower particles with energy above \( E_{\text{max}} \) and the term \( dN_{\text{ph}}/dl \) is calculated with Eq. (8) in the limit \((E_{\text{min}}/E) \to 0\). Numerical stability with negligible deviations was obtained with the choice of 10 MeV for the energy bin and 400 MeV for \( E_{\text{max}} \).

Finally, the shower axis is broken up into 200 length bins and the Cherenkov beam strength in a wavelength bin \( \Delta \lambda \) at some point on the shower axis \( x_i \) is calculated:

\[
N_{\text{ph}}(x_i, \Delta \lambda) = [\Delta N_{\text{ph}}(\Delta \lambda) + N_{\text{ph}}(x_{i-1}, \Delta \lambda)] \times \mathcal{T}(x_i, x_{i-1})
\]

(11)

where \( \mathcal{T}(x_i, x_{i-1}) \) is the atmospheric light attenuation, given by Eq. (3), between the center of the bins \( i - 1 \) and \( i \).

The angular distribution of the Cherenkov photons, \( N_{\text{ph}} \), in solid angle, \( \Omega \), is given by [14].

\[
\frac{dN_{\text{ph}}}{d\Omega} = \frac{e^{-\theta/\theta_0}}{\sin \theta}
\]

(12)

where \( \theta \) is the angle of emission of the light with respect to the shower axis, and \( \theta_0 \) is parameterized as a function of the Cherenkov threshold energy, \( E_{\text{min}} \), as follows:

\[
\theta_0 = aE_{\text{min}}^{-b}
\]

(13)

where \( E_{\text{min}} \) is in MeV, \( a = 0.83, b = 0.67 \), and the result is in units of radians.

With the Cherenkov beam strength and the angular distribution of the light, the program calculates the amount of direct Cherenkov light arriving at the detector (if any). Included in the calculation is the wavelength-dependent light attenuation between the shower and the detector, as described by Eq. (3).

Light may be scattered from the intense Cherenkov beam by the Rayleigh process. This is calculated using the molecular scattering model described in [9]. The number of photons scattered out of the beam per unit length [16] is given by

\[
\frac{dN_{\text{ph}}}{dl} = -\rho \frac{N_{\text{ph}}}{X_R} \left( \frac{\lambda}{\lambda_R} \right)^4
\]

(14)

where \( X_R \) is the Rayleigh attenuation length in g/cm\(^2\) at \( \lambda_R = 400 \) nm, \( \rho \) is the air density in g/cm\(^3\), and \( \lambda \) is the photon wavelength in nanometers.

The Rayleigh differential cross-section gives the angular distribution of Rayleigh-scattered photons as

\[
\frac{d^2N_{\text{ph}}}{dl d\Omega} = \frac{3}{16\pi} \frac{dN_{\text{ph}}}{dl} (1 + \cos^2 \theta)
\]

(15)

where \( \theta \) is the angle between the shower axis and the scattered photon. In this way the total number of photons scattered towards the detector is calculated in 20 wavelength bins for each segment of the shower track. Attenuation factors are applied according to Eq. (3).

A similar procedure is used to calculate the number of Mie-scattered Cherenkov photons arriving at the diaphragm of a telescope. As described in [9] the so-called phase function for
Mie scattering can be approximated by

\[
d^2N_{ph} \frac{d}{d\Omega} = a \exp\left(-\theta/\theta_M\right) \frac{dN_{ph}}{dl}
\]

(16)

where (parameters) \(a = 0.8\) and \(\theta_M = 26.7^\circ\). The number of Mie-scattered photons out of the beam per unit length, \(dN_{ph}/dl\), is given in the wavelength independent model by

\[
dN_{ph} \frac{dl}{l_M} = N_{ph} \exp\left(-h/h_M\right),
\]

(17)

where again \(l_M\) is the aerosol horizontal attenuation length, and \(h_M\) is the aerosol scale height.

Fig. 2 shows an example of the simulated light flux at the diaphragm with all the light components represented: fluorescence, direct Cherenkov, aerosol-scattered Cherenkov, and Rayleigh-scattered Cherenkov light. The thick solid line in the plot represents the total flux. It is expressed in terms of 370 nm-equivalent photons per time slot of 100 ns. This is the result of a simulation of a Gaisser–Hillas profile corresponding to a \(10^{19}\) eV shower landing 11,600 m from the telescope. The shower geometry has been chosen in order to favor the collection of direct Cherenkov light. The zenith angle of the shower is \(51^\circ\) and its azimuthal direction is displaced only by \(5^\circ\) from the azimuthal direction of the telescope axis. The simulation used the “clean” atmosphere parameters as described in Section 7.

The importance of the Cherenkov contribution for the Auger FD was investigated by Bellido [15] with simulations at different shower energies. As expected, because of the anisotropy in the Cherenkov light emission (Eq. (12)), the mean Cherenkov contribution is shown to be strongly dependent on the distance between the FD and the shower core. For showers at \(10^{20}\) eV, for example, it is shown that the mean Cherenkov contribution for shower core distances from 2 to 20 km decreases from about 40% to 5% of the total collected light. Those results show the importance to include the Cherenkov contamination in the present simulation program.

4. Air showers

While air shower simulation is not in the scope of FDSim, some simple shower facilities are implemented in the code. The user can elect to use the Gaisser–Hillas parameterization [4] for the shower’s longitudinal development profile (number of charged particles versus depth), or particle number and energy deposition profiles may be read from files generated by the CORSIKA program [7,10]. This choice is implemented via a datacard. For the Gaisser–Hillas profiles, the user can select specific values of energy, depth of maximum, \(X_0\) and the shower geometry, or those parameters may be randomly generated from appropriate distributions. In the case of CORSIKA simulations, the zenith and azimuth angles of the shower are already specified by CORSIKA, but the user can assign a particular core location for the event.

5. Raytracing and telescope response

The design of the Auger FD telescopes is based on Schmidt optics and it is described in detail in the prototype paper [2]. A telescope is composed of a circular aperture (the so-called diaphragm), a spherical mirror, and a \(20 \times 22\) pixel PMT camera.
The camera (focal) surface is spherical to maintain good image quality. The diaphragm contains two additional elements—an optical filter that absorbs light below 300 and above 420 nm (to reduce night sky background light), and a corrector lens. This lens does not fill the entire aperture, but forms a ring around the perimeter of the diaphragm. In the focal plane, gaps between the hexagonal PMTs are filled by the so-called Mercedes collectors which reflect light onto the active regions of the PMT [2]. Fig. 3 shows a schematic view of the telescope with the aperture system, the PMT camera and the spherical mirror.

In FDSim the telescope optics is simulated in a ray-tracing module, where all the optical components described above are taken into account. The properties of the various optical components (e.g. transmittance, reflectivity and efficiency as function of the wavelength) are parameterized in the code according to experimental measurements.

6. Electronics and trigger simulations

The signal from each FD pixel is digitized with a sampling rate of 10 MHz. The process of assigning a trigger to a pixel involves comparing a sum of ADC samples with a threshold value. Therefore, background photons play a crucial role in the simulation of the trigger. For this reason photon noise has been carefully studied and modeled. The analysis of variances of noise (i.e. not triggered) pixels has shown that

\[
\sigma_{pe}^2 = \bar{n}_{pe} (1 + V_G)
\]

(18)

where \(\sigma_{pe}^2\) is the photoelectron variance, \(\bar{n}_{pe}\) is the photoelectron mean and \(V_G\) is the gain variance factor. This equation shows that noise fluctuations can be simply viewed as originating from pure Poissonian noise due to photoelectrons (\(\sigma_P^2 = \bar{n}_{pe}\)) convolved with Gaussian noise having a mean of 0 and a variance of \(\sigma_G^2 = \bar{n}_{pe} V_G\), in order to have

![Fig. 3. Schematic view of the fluorescence telescope. From left to right can be seen the aperture system, the PMT camera and the spherical mirror.](image)
\[ \sigma_{pe}^2 = \sigma_p^2 + \sigma^2_G. \]

The gain variance factor, \( V_G \), is a parameter that represents the overall Gaussian broadening of the Poisson distribution due to the PMT dynode chain. An estimate of this factor has been obtained from PMT test measurements, giving \( 1 + V_G \approx 1.40 \). Fig. 4 shows how this model agrees with real data.

The transformation of photons arriving at the telescope to ADC-counts is done using the experimental calibration constants of the FD telescopes [17,18], which is currently performed with 370 nm light. For a given pixel \( i \), the calibration constant \( C_i \) relates the number of ADC-counts \( N_{i,\text{ADC}} \) to the number of 370 nm-equivalent photons \( N_{i,\text{370Dia}} \) arriving at the diaphragm in a given 100 ns period,

\[ N_{i,\text{370Dia}} = C_i \times N_{i,\text{ADC}}. \tag{19} \]

As previously described, FDSim calculates the arriving light in 20 wavelength bins; this light is combined into a single 370 nm-equivalent number using the measured optical efficiency of the telescopes, \( \varepsilon_\lambda \), at the 20 wavelengths. The parameters \( \varepsilon_\lambda \) take into account optical filter and corrector ring transmission, mirror reflectivity and PMT quantum efficiency. \( N_{i,\text{370Dia}} \) is then calculated:

\[ N_{i,\text{370Dia}} = \frac{1}{\varepsilon_{370}} \sum_{\lambda} N_{i,\varepsilon_\lambda}. \tag{20} \]

where \( N_{i,\varepsilon_\lambda} \) is the number of photons in a wavelength bin arriving at the diaphragm in a 100 ns period.

Once the ADC counts have been calculated, the trigger simulation proceeds in two steps corresponding to the two hardware implemented trigger levels [19].

The First Level Trigger (FLT) is an algorithm which applies to each pixel and is based upon the running sums of the last 10 samples. A pixel is marked as triggered, whenever the running sum exceeds an adjustable threshold. When the running sum again falls below the threshold, the pixel trigger is extended by a time of 20 \( \mu s \), which is the overlap coincidence time common to all pixels. The trigger rate of individual pixels is kept close to a reference value of 100 Hz by automatic adjustment of the threshold. The trigger hardware has been implemented through reprogrammable FPGA logic. The number of samples of the running sum, the trigger time overlap and the reference trigger rate are programmable, allowing high flexibility.

The simulation code strictly follows this trigger logic. Two parameters control the FLT in the process: the trigger rate, by default set at 100 Hz, which is used to calculate the threshold, and the number of samples, by default set at 10, over which ADC summation is performed to give running sums. The program calculates the threshold through interpolation of the reduced threshold:

\[ t_N = \frac{T_N^{\text{ADC}} - B_N^{\text{ADC}}}{\sigma_N^{\text{ADC}}}, \tag{21} \]

where \( T_N^{\text{ADC}}, B_N^{\text{ADC}} \) and \( \sigma_N^{\text{ADC}} \) are the threshold, baseline and rms, respectively, in ADC counts of running sums over \( N \) samples. Reduced thresholds are calculated analytically for different values of the random noise, \( \bar{n}_{pe} \), and of the gain variance factor, \( V_G \).

The Second Level Trigger (SLT) is also simulated according to the detector specifications. The SLT algorithm searches for five pixels, within a
defined time gate, shaped as pre-defined straight segments. These patterns are derived from fundamental patterns and those created by rotation and mirror reflections. The algorithm accepts patterns even where one pixel out of the five has not triggered, allowing for possible faulty PMTs or small signals below the FLT threshold. As a whole 108 patterns classes are distinguished.

The simulation reproduces the pipeline structure of the SLT FPGA implemented in the hardware trigger. The pipeline holds the image of a $22 \times 5$ submatrix of the camera. The search for the allowed patterns is performed inside this submatrix, then the image is shifted, column-by-column, over the full camera. Twenty cycles complete the full camera scan (1 µs real time).

The simulation output is translated at the end of the simulation process into the same format used by the FD data acquisition system, and it is streamed to a file for storage. Fig. 5 shows the simulated event of the example of Section 3.3 visualized with the online event display used in the acquisition system of the FD.

7. Trigger efficiency

In this section, we present the result of an application of FDSim in calculating the trigger efficiency of the southern observatory fluorescence detectors. The four fluorescence sites currently under construction are shown in Fig. 6. Each site (or “eye”) houses six telescopes, and covers an azimuth range of $180^\circ$ and an elevation range of approximately $30^\circ$. The position of the telescopes and their pointing directions, are set by datacards in the simulation, making it a useful package for northern observatory design studies planned in the near future.

The simulations described here have produced hybrid trigger efficiencies for showers falling over the full southern site for the energy range $10^{17.5} \text{–} 10^{20} \text{eV}$ and zenith angles from $0^\circ$ to $60^\circ$. A hybrid trigger is defined as a trigger in at least one FD eye and at least one surface detector (SD) tank. The single-tank SD trigger probability is essentially unity for the energies and zenith angles considered here.

![Fig. 5. The on-line event display of the FD data acquisition system is used here to display the simulated event of Section 3.3. In the upper left diagram, the pixel matrix with triggered pixels is shown. The pixels with a black dot are selected for the ADC view. In the upper right diagram, the ADC values of the selected pixels are shown as a function of time in 100 ns bins. The lower right diagram shows the trigger intervals for each camera column as a function of time in bins of 1 µs.](image-url)
Gaisser–Hillas shower profiles were used in the simulation. The datacards were setup so that the depths of maximum of the showers were selected from distributions based on CORSIKA simulations of proton air showers using the QGSJET hadronic model [20]. FDSim was run, and two post-simulation cuts were applied to ensure reconstructability: the angular track length at the eye must be \( \frac{4}{6} \) and the eye must view shower maximum.

Fig. 7 shows the result of the simulations. Two different atmospheric aerosol concentrations were assumed. These are labelled “clean” and “dirty” atmospheres. In both cases the aerosol scale height was set at 1200 m; the sea-level horizontal aerosol attenuation length was set to 11,700 m (clean) or 4700 m (dirty). (These correspond to horizontal attenuation lengths at the detector altitude (1400 m) of 38,000 and 15,000 m, respectively). We expect many more “clean” nights of observation than “dirty” ones.

The trigger efficiency is shown in Fig. 7 for showers triggering at least one, two, three or four FD eyes. This is converted to an aperture assuming the total available aperture of 7350 km\(^2\)sr for the zenith angle range 0–60°. The plot confirms the design goal of near full efficiency for showers above \(10^{19}\) eV; it indicates that a large aperture is available even at lower energies, and it shows that more than half of events seen above \(10^{19}\) eV will be seen by more than one eye. Multi-fold FD station triggers clearly dominate above this energy.

8. Conclusions

We have presented details of the FD simulation code used by the Pierre Auger Observatory. It is a flexible and modular package which simulates the production of fluorescence and Cherenkov light by air showers, the transmission of that light to the detector and the optics and light collection by the detector telescopes. The simulation of the electronics and trigger completes the package. We have demonstrated the use of FDSim in detection efficiency calculations.
FDSim is continually evolving as more sophisticated treatments of various parts of the physics are introduced. The structure is also evolving, since it will soon become part of the collaboration’s offline software framework. This will merge analysis and simulation software for both surface and fluorescence detectors.

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