Solution of transient direct-chill aluminium billet casting problem with simultaneous material and interphase moving boundaries by a meshless method

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Abstract

This paper uses a recently developed upgrade of the classical meshless Kansa method for solution of the transient heat transport in direct-chill casting of aluminium alloys. The problem is characterised by a moving mushy domain between the solid and the liquid phase and a moving starting bottom block that emerges from the mould during the process. The solution of the thermal field is based on the mixture continuum formulation. The growth of the domain and the movement of the bottom block are described by activation of additional nodes and by the movement of the boundary nodes through the computational domain, respectively. The domain and boundary of interest are divided into overlapping influence areas. On each of them, the fields are represented by the multiquadrics radial basis function collocation on a related sub-set of nodes. Time stepping is performed in an explicit way. The governing equation is solved in its strong form, i.e. no integrations are performed. The polygonisation is not present and the method is practically independent of the problem dimension. Realistic boundary conditions and temperature variation of material properties are included. An axisymmetric transient test case solution is shown at different times and its accuracy is verified by comparison with the reference finite volume method results.

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1. Introduction

Direct-chill (DC) casting (Fig. 1) is currently the most common \cite{1} semi-continuous casting practice in production of aluminium alloys. The process involves molten metal being fed through a bottomless water-cooled mould where it is sufficiently solidified around the outer surface to take the shape of the mould and acquire sufficient mechanical strength to contain the molten core at the centre. As the strand emerges from the mould, water impinges directly from the mould onto the surface (DC), flows over the cast surface and completes the solidification. Related transport, solid mechanics, and phase change kinetics phenomena are extensively studied \cite{2}. A technically crucial phase of the DC casting process is the start-up (or initial) phase, a period from the start of operation until a steady state. The DC casting start-up phase is short (several minutes) compared to the whole casting (several hours). In this phase, the starting block, which is initially in the mould (and carries the whole weight of the ingot throughout the process) starts to move downwards. The steady-state operation is approximately achieved when the starting block moves away from the mould for a few typical transversal dimensions of the cast. Although short, this phase is of critical importance for the quality of the final product. In addition to the moving interphase boundary, the consideration of the start-up phase involves a moving...
A common complexity of the classical numerical methods is the need to create a polygonisation, either in the domain and/or on its boundary. This type of (re)meshing is often the most time-consuming part of the solution process and is far from being fully automated, particularly in moving and/or deforming boundary problems. In recent years, a new class of methods is in development, which do not require polygonisation but use only a set of nodes to approximate the solution. The rapid development of these types of meshless (polygon-free) methods and their classification is elaborated in the very recent monographs [4,5].

A broad class of meshfree methods in development today are based on radial basis functions (RBFs) [6]. The global RBF collocation method (RBFCM) or Kansa method [7] is the simplest of them. The method has been extended to cope with large-scale diffusion problems in [8], and to convective–diffusive solid liquid phase change problems in [9] by developing its local version (LRBFCM). Two test cases have been elaborated in [8]. The first is the boundary value problem, associated with the Dirichlet jump problem on a square. The accuracy of the method is assessed in terms of the average and maximum errors with respect to the density of the nodes, number of nodes in the domain of influence, multiquadrics free parameter, and time step length on uniform and non-uniform node arrangements. The LRBFCM outperforms the classical finite difference method in terms of accuracy for the same number of nodes in all situations except immediately after the Dirichlet jump where the approximation properties appear similar. Several test cases have been performed in [9] for problems with different Stefan and Péclet numbers as well as uniform and non-uniform randomly displaced node arrangements. Excellent agreement with the analytical solution and reference solutions has been found. The LRBFCM is suitable for problems where sharp gradients appear, such as the phase field model [10], since it is relatively very simple to refine the computational nodes without additional remeshing.

Many different numerical methods have been used in the past to solve the related DC casting models. Among them are also several mesh reduction and meshless techniques. The boundary element method for heat transport has been used in [11], the dual reciprocity boundary element method in [12] and its extension with fluid flow in [13]. The use of the meshless Kansa method has been pioneered in [14]. The use of the diffuse approximate method has been implemented in [15,16] and the use of LRBFCM in [9]. This paper upgrades the recently developed LRBFCM to solve the heat transfer model of the start-up phase of the DC casting. A principal complication in the present effort is a proper adding of the boundary points and a proper omission of the nearby domain nodes near the moving boundary of the system.

2. Governing equations

Consider a connected domain $\Omega$ with boundary $\Gamma$, occupied by a liquid–solid phase change material described with the temperature-dependent density $\rho_\varphi$ of the phase $\varphi$, temperature-dependent specific heat at constant pressure $c_\varphi$, thermal conductivity $k_\varphi$, and the specific latent heat of

![Fig. 1. Scheme of the start-up phase of the DC casting process at different times.](image-url)
the solid–liquid phase change \( h_m \). The domain and boundary are assumed to grow with time \( t \), i.e., \( \Omega(t_0) \subseteq \Omega(t_0 + \Delta t) \), \( \Gamma(t_0) \subseteq \Gamma(t_0 + \Delta t) \), where \( t_0 \) represents initial time and \( \Delta t \) the positive time increment. The one-phase continuum formulation of the enthalpy conservation for the assumed system is
\[
\frac{\partial}{\partial t} (\rho\overline{h}) + \nabla \cdot (\rho\overline{v}\overline{h}) = \nabla \cdot (k \nabla T) \\
+ \nabla \cdot (\rho\overline{v} - f^V_S \rho_S \overline{v}_S - f^V_L \rho_L \overline{v}_L), \tag{1}
\]
where the second term on the right-hand side is a correction term, needed to accommodate the mixture formulation of the convective term. The mixture density and thermal conductivity are defined as
\[
\rho = f^V_S \rho_S + f^V_L \rho_L, \tag{2}
\]
\[
k = f^V_L k_L, \tag{3}
\]
where \( f^V_\phi \) represents the volume fraction of phase \( \phi \).
The liquid volume fraction \( f^V_L \) is assumed to vary linearly from 0 to 1 between solidus \( T_S \) and liquidus temperature \( T_L \). The mixture velocity and mixture enthalpy are defined as
\[
\overline{v} = (f^V_S \overline{v}_S + f^V_L \overline{v}_L) / \rho, \tag{4}
\]
\[
\overline{h} = (f^V_S \rho_S \overline{v}_S + f^V_L \rho_L \overline{v}_L) / \rho. \tag{5}
\]
The constitutive temperature–enthalpy relationships are
\[
h_S = \int_{t_{ref}}^T \overline{c}_S \, dT, \tag{6}
\]
\[
h_L = h_S(T) + \int_{T_{ref}}^T (c_L - c_S) \, dT + h_m, \tag{7}
\]
with \( T_{ref} \) standing for the reference temperature. Thermal conductivity and specific heat of the phases can arbitrarily depend on temperature. We seek for the mixture temperature at time \( t_0 + \Delta t \) by assuming a known initial temperature, velocity field, and boundary conditions at time \( t_0 \). The initial value of the temperature \( T(p, t) \) at a point with position vector \( p \) and time \( t_0 \) is defined through the known function \( T_0 \)
\[
T(p, t_0) = T_0(p); \quad p \in \Omega(t_0) + \Gamma(t_0). \tag{8}
\]

The boundary \( \Gamma \) is divided into not necessarily connected parts \( \Gamma(t) = \Gamma^D(t) \cup \Gamma^R(t) \cup \Gamma^N(t) \) with Dirichlet, Neumann and Robin type boundary conditions, respectively. At the boundary point \( p \) with normal \( n_r \) and time \( t_0 \leq t \leq t_0 + \Delta t \), these boundary conditions are defined through known functions \( T^D_r, T^N_r, T^R_r, T^R_{ref} \)
\[
T = T^D_r; \quad p \in \Gamma^D, \tag{9}
\]
\[
\frac{\partial}{\partial n_r} T = T^N_r; \quad p \in \Gamma^N, \tag{10}
\]
\[
\frac{\partial}{\partial n_r} T = T^R_r; \quad p \in \Gamma^R. \tag{11}
\]

2.1. Solution procedure

The representation of function \( \Phi \) over a set of \( l \) (in general) non-equally spaced \( \mathcal{N} \) nodes \( \mathbf{p}_n ; \quad n = 1, 2, \ldots, \mathcal{N} \) is made in the following way:
\[
\Phi(p) \approx \sum_{k=1}^{\mathcal{K}} \psi_k(p) / \gamma_k. \tag{12}
\]
\( \psi_k \) stands for the shape functions, \( \gamma_k \) for the coefficients of the shape functions, and \( \mathcal{K} \) represents the number of the shape functions. The left lower index on entries of expression (12) represents the domain of influence \( r_{\omega} \) on which the coefficients \( \gamma_k \) are determined. The domains of influence \( r_{\omega} \) can in general be contiguous (overlapping) or non-contiguous (non-overlapping). Each of the domains of influence \( r_{\omega} \) includes \( \mathcal{N} \) grid points of which \( \mathcal{N} \) are in the domain and \( \mathcal{N} \) are on the boundary. The coefficients can be calculated from the domain of influence nodes in two distinct ways. The first way is collocation (interpolation) and the second way is approximation by the least squares method. Only the simpler collocation version for calculation of the coefficients is considered in this text. Let us assume the known function values \( \Phi_n \) in the nodes \( \mathbf{p}_n \) of the domains of influence \( r_{\omega} \). The collocation implies
\[
\Phi(p) = \sum_{k=1}^{\mathcal{N}} \psi_k(p) / \gamma_k. \tag{13}
\]

For the coefficients to be computable, the number of the shape functions has to match the number of the collocation points \( \mathcal{K} = \mathcal{N} \), and the collocation matrix has to be non-singular. The system of equations (13) can be written in a matrix-vector notation
\[
\left[ \psi \right] \mathbf{\alpha} = \mathbf{\Phi}; \quad \left[ \psi \right] \mathbf{\alpha} = \left[ \psi \right]_{\mathbf{p}_n} \mathbf{\gamma} \mathbf{\Phi}_n, \quad \left[ \psi \right] \mathbf{\Phi} = \left[ \psi \right] \mathbf{\Phi}_n. \tag{14}
\]

The coefficients \( \mathbf{\alpha} \) can be computed by inverting the system (14)
\[
\mathbf{\alpha} = \left[ \psi \right]^{-1} \mathbf{\Phi}. \tag{15}
\]

By taking into account the expressions for the calculation of the coefficients \( \mathbf{\alpha} \), the collocation representation of function \( \Phi(p) \) on domain of influence \( r_{\omega} \) can be expressed as
\[
\Phi(p) \approx \sum_{k=1}^{\mathcal{N}} \psi_k(p) \sum_{n=1}^{\mathcal{N}} \psi_k^{-1}(\mathbf{p}_n) \gamma_n, \quad \mathbf{\gamma} = r_{\omega}. \tag{16}
\]

Let us introduce a two-dimensional Cartesian coordinate system with base vectors \( i_z, \gamma = r, \zeta \) and coordinates \( p, \gamma = r, \zeta \) and coordinates \( p, \zeta = r, z \), i.e. \( p = i_r p_r; \quad \zeta = r, z \). The first partial spatial derivatives of \( \Phi(p) \) on domain of influence \( r_{\omega} \) can be expressed as
\[
\frac{\partial}{\partial p_r} \Phi(p) \approx \sum_{k=1}^{\mathcal{N}} \frac{\partial}{\partial p_r} \psi_k(p) \sum_{n=1}^{\mathcal{N}} \psi_k^{-1}(\mathbf{p}_n) \gamma_n, \quad \mathbf{\gamma} = r, \zeta. \tag{17}
\]
The second partial spatial derivatives of \( \Phi(p) \) on domain of influence \( \Omega \) can be expressed as
\[
\frac{\partial^2 \Phi(p)}{\partial p, p}\zeta \approx \sum_{k=1}^{N} \frac{\partial^2 \phi\zeta}{\partial p, p}\zeta r_k(p) \sum_{\eta=1}^{N} \psi^{-1}_{k\eta} \Psi_{nn}; \quad \zeta = r, z.
\]

The RBFs, such as multiquadrics, can be used for the shape function
\[
\phi_k(p) = [1 + c^2 r_k^2 / r_0^2]^{1/2}; \\
\psi_k^2 = (p - p_k) \cdot (p - p_k),
\]
where \( c \) represents the shape parameter and \( r_0 \) the scaling constant, set to the maximum distance between two nodes in a domain of influence. What follows elaborates the solution of the convection–diffusion equation (1), subject to the initial condition (8), and the boundary conditions (9–11). The temperature can be iteratively expressed from the enthalpy as
\[
h = h + \frac{dh}{dT} (T - T).
\]

The unknown function value \( T_l \) in domain node \( p_j \) can be calculated as
\[
T_l = \left\{ \frac{h_0 + \frac{dh}{dT} l T_l - h_l + \frac{\Delta t}{\rho_0 c_0}}{\rho_0 c_0 l} \left[ -\nabla \cdot (c_0 v_0 h_0) + \nabla k_0 \cdot \nabla T_l + k_{\text{ld}} \cdot \nabla^2 T_l \right] \right\} / \frac{dh}{dT} l T_l.
\]

The described formulation and handling of the governing equation belongs to the class of fixed-grid or one-domain techniques for phase change problems. The explicit calculation of expression (22) in axisymmetry is
\[
T_l = \left\{ h_0 + \frac{dh}{dT} l T_l - h_l \right\} + \frac{\Delta t}{\rho_0 c_0 l} \left[ \sum_{k=1}^{N} \nabla \cdot (c_0 v_0 h_0) \psi^{-1}_{k\eta} \Psi_{nn} \right] + \frac{\Delta t}{\rho_0 c_0 l} \left[ \sum_{k=1}^{N} \nabla \cdot (c_0 v_0 h_0) \psi^{-1}_{k\eta} \Psi_{nn} \right] \times \left[ \sum_{n=1}^{N} \nabla \cdot (c_0 v_0 h_0) \psi^{-1}_{k\eta} \Psi_{nn} \right].
\]
where the formulas (17) and (18) have been employed. The complete solution procedure follows steps 1–6 defined below.

Step 1: First, the initial conditions are set in the domain and boundary nodes and the required derivatives are calculated from the known nodal values.

Step 2: Eq. (23) is used to calculate the new values of the variable \( lT_n \) at time \( t_0 + \Delta t \) in the domain nodes.

Step 3: What follows in steps 3 and 4, defines variable \( lT_n \) at time \( t_0 + \Delta t \) in the Dirichlet, Neumann, and Robin boundary nodes of the new boundary at time \( t_0 + \Delta t \). For this purpose, in step 3, the coefficients \( l_a \) have to be determined from the new values in the domain \( O(t_0 + \Delta t) \) and from the information on the boundary conditions at \( \Gamma(t_0 + \Delta t) \). Let us introduce domain, Dirichlet, Neumann, and Robin boundary indicators for this purpose. These indicators are defined as

\[
\begin{align*}
\gamma_{\Omega} & = \begin{cases} 1 \text{; } p_n \in \Omega, \\ 0 \text{; } p_n \notin \Omega \end{cases}, \quad \gamma_{\Gamma}^D = \begin{cases} 1 \text{; } p_n \in \Gamma^D, \\ 0 \text{; } p_n \notin \Gamma^D \end{cases}, \\
\gamma_{\Gamma}^N & = \begin{cases} 1 \text{; } p_n \in \Gamma^N, \\ 0 \text{; } p_n \notin \Gamma^N \end{cases}, \quad \gamma_{\Gamma}^R = \begin{cases} 1 \text{; } p_n \in \Gamma^R, \\ 0 \text{; } p_n \notin \Gamma^R \end{cases}.
\end{align*}
\]

The coefficients \( l_a \) are calculated from the system of linear equations

\[
\begin{align*}
\sum_{k=1}^{iN} \gamma_{\Omega n} \frac{\partial}{\partial \tau} \psi_k(p_n) z_k + \sum_{k=1}^{iN} \gamma_{\Gamma n}^D \frac{\partial}{\partial \tau} \psi_k(p_n) z_k \\
+ \sum_{k=1}^{iN} \gamma_{\Gamma n}^N \frac{\partial}{\partial \tau} \psi_k(p_n) z_k + \sum_{k=1}^{iN} \gamma_{\Gamma n}^R \frac{\partial}{\partial \tau} \psi_k(p_n) z_k \\
= \gamma_{\Omega n} lT_n \Gamma + \gamma_{\Gamma n}^D lT_n^D + \gamma_{\Gamma n}^N lT_n^N + \gamma_{\Gamma n}^R lT_n^R \\
\times \left( \sum_{k=1}^{iN} \psi_k(p_n) z_k - i lT_{\text{ref}} \right).
\end{align*}
\]  

The system (25) can be written in a compact form

\[
\begin{align*}
\psi / \alpha &= b.
\end{align*}
\]  

Fig. 4. Distribution of points at times \( t = 500, 1000, \) and 1500 s for the coarser grid-point arrangement I.
with the following system matrix entries:
\[
\begin{aligned}
&i\mathbf{P}_{nk} = i Y_{nk} \psi_k(\mathbf{p}_n) + i Y^D_{nk} \psi_k(\mathbf{p}_n) \\
&+ i Y^N_{nk} \frac{\partial}{\partial n} \psi_k(\mathbf{p}_n) + i Y^R_{nk} \\
&\times \left[ \frac{\partial}{\partial n} \psi_k(\mathbf{p}_n) - i T^N_{nk} \sum_{k=1}^N \psi_k(\mathbf{p}_n) \right],
\end{aligned}
\]
(27)

and with the following explicit form of the augmented right-hand side vector:
\[
\mathbf{b}_n = i Y_{nk} T_n + i Y^D_{nk} T^D_n + i Y^N_{nk} T^N_n - i Y^R_{nk} T^R_n \\
+ i Y^R_{nl} T^R_{nl} + i Y^R_{ref} T^R_{ref}.
\]
(28)

Step 4: The bottom boundary nodes are moved according to the casting velocity and time step length
\[
\Delta s_l = v_{el} \cdot \Delta t.
\]
(29)

Step 5: The unknown boundary values of \(\Gamma(t_0 + \Delta t)\) are set from Eq. (16). When the distance between the moving boundary nodes and the fixed domain exceeds two times the typical grid distance of the node arrangement, new inner nodes are inserted between moving boundary nodes and inner nodes. Their values are obtained by the LRBF interpolation (Eq. (16)). The growth of the domain and the respective node manipulations are represented in Fig. 2.

Step 6: The under-relaxation
\[
T_n = T_n^{relax} + \varepsilon (T_n - T_n^{relax})
\]
might be required in the general case for all the computational nodes \(\mathbf{p}_n; \ n = 1, 2, \ldots, N\), with \(\varepsilon\) standing for the under-relaxation factor. The iterations over one time step are completed when the iteration criterion (30)
\[
\max |T_n - T_{on}| \leq T_{itr},
\]
(30)
\[
\max |T_n - T_{on}| \leq T_{ste},
\]
(31)
is satisfied in all computational nodes. The steady state is achieved when criterion (31) is met. The parameter \(T_{ste}\) is defined as the steady-state convergence margin. In case the steady-state criterion is achieved or the time of calculation exceeds the foreseen time of interest, the calculation is stopped (Fig. 3).

3. Numerical example

This section elaborates the solution of a simplified model of the start of the DC casting process by the developed LRBF in axisymmetry. The transient solution is calculated using a fixed time step of 0.1 s. The temperature iteration error \(T_{itr}\) has been set to 0.001 K and the steady-state criterion \(T_{ste}\) to 0.01 K (the steady state is actually never reached in represented example). The multiquadrics
free parameter was set to $c = 32$ in accordance with the study in [8]. The enthalpy reference temperature $T_{ref}$ has been set to 0 K. The following simplified DC casting case is considered. The initial computational domain is a cylinder (coordinates $p_z, p_r$) $-0.01 \text{ m} \leq p_z \leq 0 \text{ m}, 0 \text{ m} \leq p_r \leq 0.25 \text{ m}$. The boundary conditions on the top at $p_z = 0 \text{ m}$ are of the Dirichlet type with $T_D = 980 \text{ K}$, and the boundary conditions at the moving bottom are of the Neumann type with $F_N = 0 \text{ W/m}^2$. The boundary conditions at the outer surface are of the Robin type with $T_R = 298 \text{ K}$. The heat transfer coefficients between $0 \text{ m} \leq p_z \leq -0.01 \text{ m}, -0.01 \text{ m} < p_z < 0.06 \text{ m}, -0.06 \text{ m} < p_z < 0.1 \text{ m}$, and $-0.1 \text{ m} < p_z < -1.25 \text{ m}$, are $T_f^R = 0 \text{ W/m}^2 \text{ K}$, $T_f^R = 3000 \text{ W/m}^2 \text{ K}$, $T_f^R = 150 \text{ W/m}^2 \text{ K}$, and $T_f^R = 4000 \text{ W/m}^2 \text{ K}$, respectively. Material properties correspond to a simplified Al 4.5% Cu alloy [12]: $\rho_S = \rho_L = 2982 \text{ kg/m}^3$, $k_S = 120.7 \text{ W/mK}$, $k_L = 57.3 \text{ W/mK}$, $c_S = 1032 \text{ J/kgK}$, $c_L = 1179 \text{ J/kgK}$, $h_M = 348.2 \text{ kJ/kgK}$, $T_S = 775 \text{ K}$, $T_L = 911 \text{ K}$. The liquid fraction increases linearly between $T_S$ and $T_L$. The initial temperature is 980 K and uniform in the initial computational domain. The uniform casting velocity is $\nu_S = \nu_L = -0.000633 \text{ m/s}$, $\nu_S = \nu_L = 0 \text{ m/s}$. The solution has been obtained with two different node arrangements that are equidistant in $p_r$ direction. The coarser one (denoted as I) includes 25 nodes in $p_r$ direction, and the finer one (denoted as II) includes 50 nodes, respectively. The domain nodes that are kept after moving the bottom (see Step 5 of the computational procedure) have the same grid distance as the nodes in $p_r$ direction, corresponding to a final number of 125 nodes (I) and 250 nodes (II) in the $p_z$ direction. The corresponding discretization is shown in Fig. 4. Each of the domains of influence contains 5 nodes (see Fig. 3). The calculated results for times $t = 500, 1000,$

![Fig. 6. Comparison of centreline (top), mid-radius (middle), and surface (bottom) temperatures calculated by the LRBFCM (solid line) with the reference FVM solution (dashed line) at times $t = 500, 1000,$ and 1500 s. The thin lines represent the steady-state solution, calculated by the LRBFCM. The left column represents calculations with coarser LRBFCM node arrangement I, and the right one with the finer one II.](image-url)
Fig. 6 shows centreline, mid-radius and surface temperatures, together with the reference finite volume method (FVM) solution, calculated in the same nodes. The finite volume model [17] employs the central-difference discretization scheme for convection–diffusion and is thus second-order accurate, which was clearly verified by calculating the observed order of convergence over three grids (25 × 125, 50 × 250 and 100 × 500 nodes). The FVM results obtained on the finer two grids were extrapolated using Richardson extrapolation to obtain a reference solution. Its error was estimated by the maximum local grid convergence index (GCI) [18] in the temperature field and is GCI < 0.1% everywhere. By a conservative estimate, this corresponds to an error band of 1 K for the ingot surface and 0.1 K elsewhere in the domain. One can observe a slight difference between LRBFCM and FVM results in Fig. 6 (left) where the node arrangement I is used and practically no difference in Fig. 6 (right) where the node arrangement II is used.

In the comparison of the numerical results obtained by the LRBFCM with the extrapolated FVM solution (reference), one can observe very good agreement. The absolute temperature difference between the LRBFCM solution with node arrangements I and II and the reference solution is shown in Fig. 7. The comparison was made by RBF interpolation of the meshless solution in FVM points. The seemingly large differences that occur in a few points are attributed to very small longitudinal shifts of the temperature profile in regions of large gradients. Identifying the source of this discrepancy and noting that everywhere else the LRBFCM solution is well inside the error band of the reference, we can conclude a very good accuracy performance of the meshless method and the proposed computation strategy.

![Fig. 7. The absolute temperature difference between the reference FVM and LRBFCM solutions at times t = 500, 1000, and 1500 s. Lines represent: solid—surface temperature, dashed—mid-radius temperature and dash-dot—centreline temperature. The left column represents calculations with coarser LRBFCM node arrangement I, and the right one with the finer one II.](image-url)
4. Conclusions

This paper represents a solution of the transient thermal field of the start-up phase of the DC casting of aluminium by the explicit LRBFCM. It demonstrates the suitability of the present meshless method to cope with complicated moving boundary problems. The governing mixture continuum equation for a solid–liquid phase change system is solved in its strong form on a moving domain, which most probably represents the first application of the present method in simultaneous material and interphase moving boundary problems. An original strategy of dealing with the growing computational domain in the context of meshless methods is proposed. It is shown to be robust and accurate. The developments are almost independent of the problem dimension. The complicated geometry is easy to cope with. No polygonisation is needed. No integrations are needed. The method appears efficient, because it does not require a solution of the large systems of equations like the original Kansa method. Instead, small systems of linear equations have to be solved in each time step for each node and associated domain of influence, representing the most natural and automatic domain decomposition. The extension of the method to cope with the flow of the liquid metal in the melt is underway, as well as simulation of the development of the stress and deformation. In this respect, the formulation, developed in [19,20] will be used.

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References