

# Advanced Numerical Methods for Engineering Applications

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May 17, 2010

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# Easy name to remember



UNLV

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University of Nevada, Las Vegas



# Various Techniques

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- ◆ High-order FDM, low-order FVM
- ◆ Chapeau functions (Pade, cubic splines)
- ◆ Method of Moments (subgrid scale)
- ◆ FEM-Petrov-Galerkin (SUPG)
- ◆ h-, p-, and hp-adaptation
- ◆ Spectral elements
- ◆ BEM
- ◆ Meshless Methods
- ◆ Stochastic Methods

# Sources of Error

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- ◆ **Physical approximation error (simplification)**
  - ✘ Physical modeling error
  - ✘ Geometry modeling error
- ◆ **Computer round-off error (32 vs 64 bit)**
- ◆ **Iterative convergence error ( $10^{-4}$ )**
- ◆ **Discretization error (mesh resolution)**
  - ✘ Spatial discretization error
  - ✘ Temporal discretization error
- ◆ **Computer programming error (bugs)**
- ◆ **Usage error (conceptual-I/O)**

# Multiple Projects

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- ◆ Atmospheric Transport and Diffusion
- ◆ Groundwater Dispersion (YMP)
- ◆ Wind Energy (site assessment)
- ◆ h-, p-, hp-adaptive CHT (KIVA code)
- ◆ IAQ
- ◆ Solar-powered UAV; thin-film
- ◆ Lunar – Mars Habitats; Haiti reconstruction
- ◆ High-speed train (LV-Victorville, CA)
- ◆ Data center cooling
- ◆ Telemedicine

# Adaptation Methodology

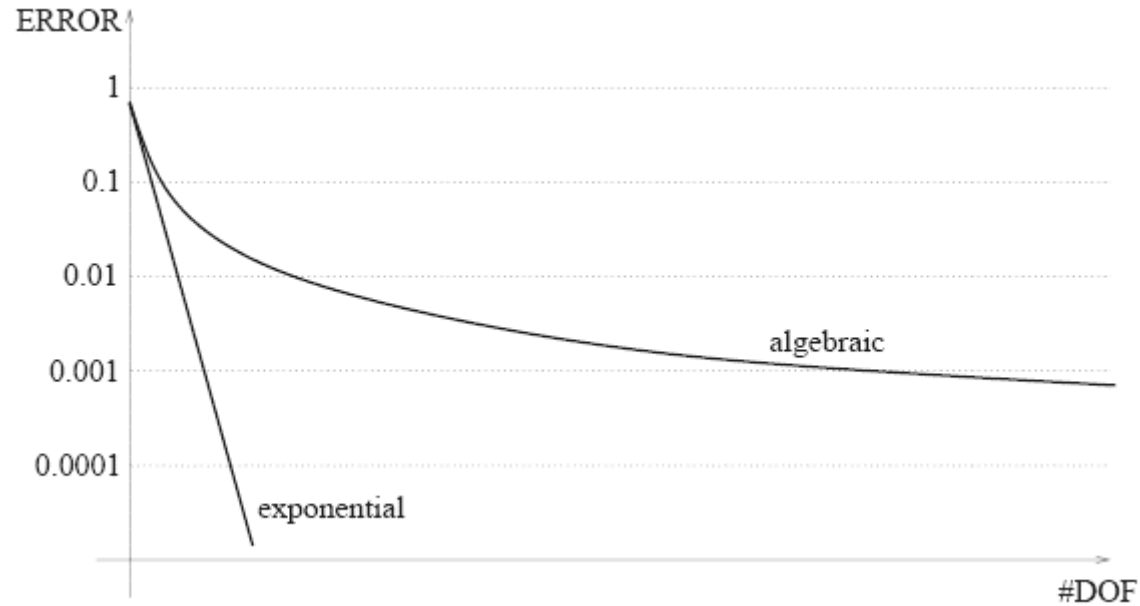
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- ◆ Various types of adaptation

- ☒ r-adaptation (fixed density)
- ☒ h-adaptation (mesh refinement)
- ☒ p-adaptation (increasing order)
- ☒ h-p adaptation (combination)

# Convergence Rates

## ◆ Exponential vs. Algebraic rate



*hp*-adaptation has the potential of converge exponentially

# Three-step hp-adaptation strategy

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- ◆ **Step 1:**

- ✘ Construct initial coarse mesh, preset target value for error

- ◆ **Step 2:**

- ✘ Construct the intermediate h-adaptive mesh

- ◆ **Step 3:**

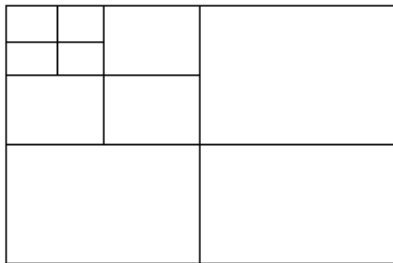
- ✘ Apply p-adaptive enrichments on the intermediate mesh to obtain the final hp adaptive mesh.



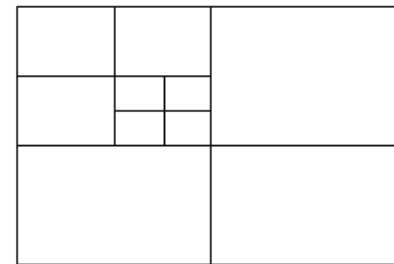
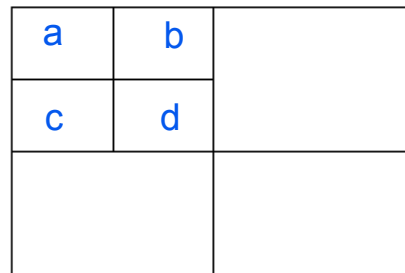
# Element rules

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- ◆ An element may be refined only if its neighbors are at the same or higher level (1-Irregular mesh)



Right

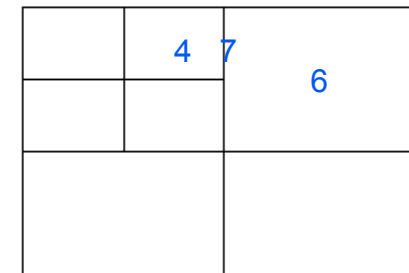
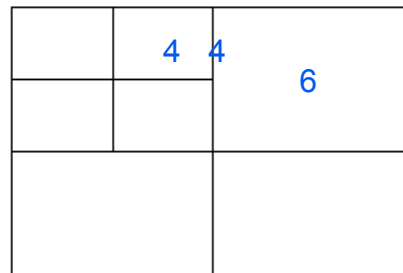
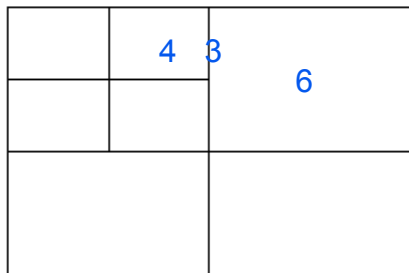


Wrong!

# p-adaptation rule

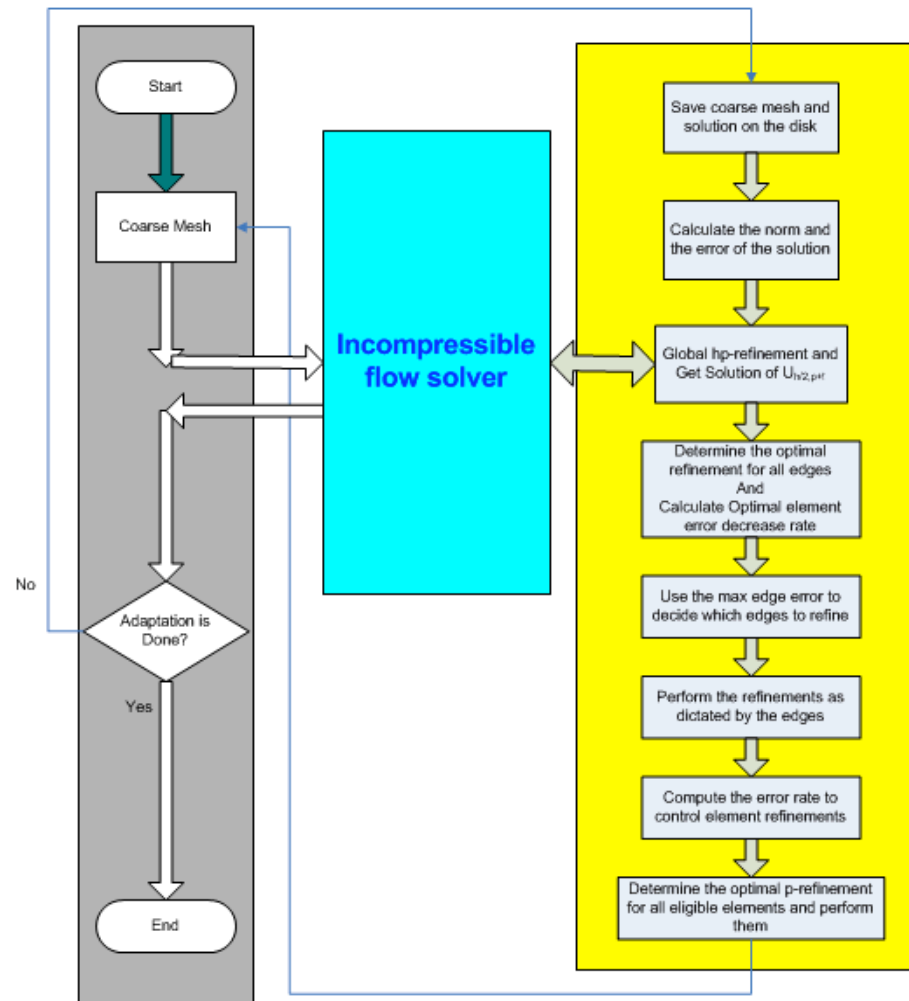
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- ◆ **Minimum rule must be followed**
  - ✎ **The order for the edges never exceeds orders of the neighboring middle nodes**



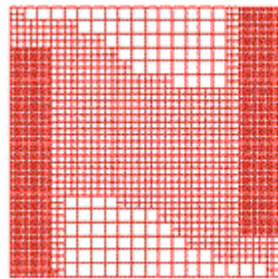
Note: the order for mid face is showed for horizontal direction only, the order for vertical direction will follow the same rule

# Fully automatic hp-strategy

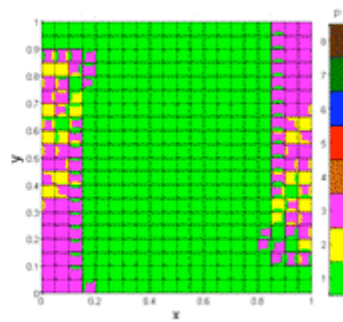


# Simulation Results

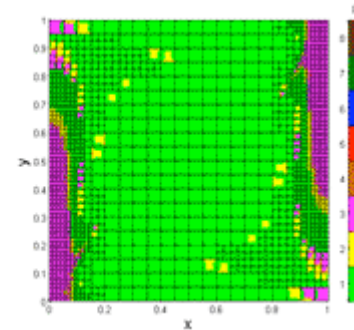
## ◆ Natural convection within cavity $Ra=10^5$



(a)  $h$ -adaptation

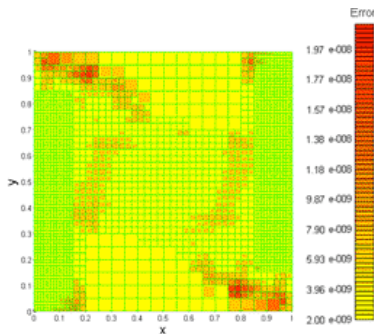


(b)  $p$ -adaptation

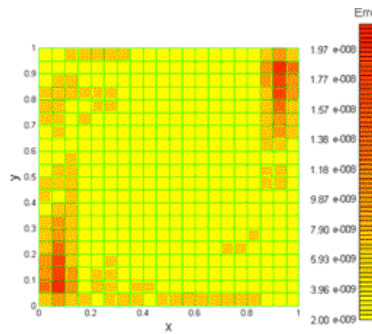


(c)  $hp$ -adaptation

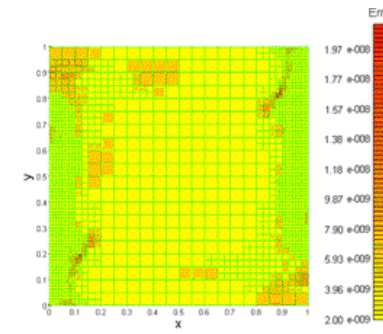
Final adaptive meshes



(a)  $h$ -adaptation



(b)  $p$ -adaptation



(c)  $hp$ -adaptation

Error distributions

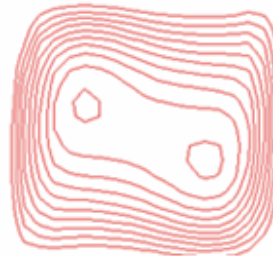
# Simulation Results

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## ◆ Natural convection within cavity –cont.

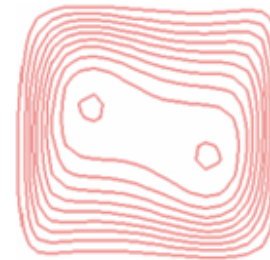


(a) *h*-adaptation

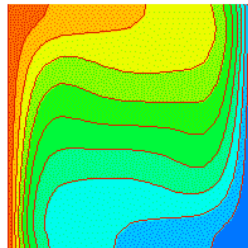


(b) *p*-adaptation

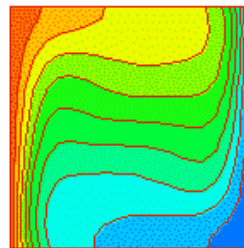
Streamfunction contours



(c) *hp*-adaptation

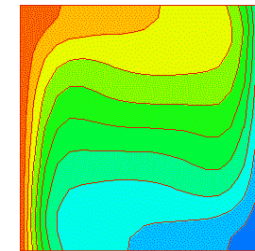


(a) *h*-adaptation



(b) *p*-adaptation

Temperature contours



(c) *hp*-adaptation

# Meshless method

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- ◆ No mesh required (optimal placement)
- ◆ Utilizes radial basis functions

$$r_j = \sqrt{(x - x_j)^2 + (y - y_j)^2}.$$

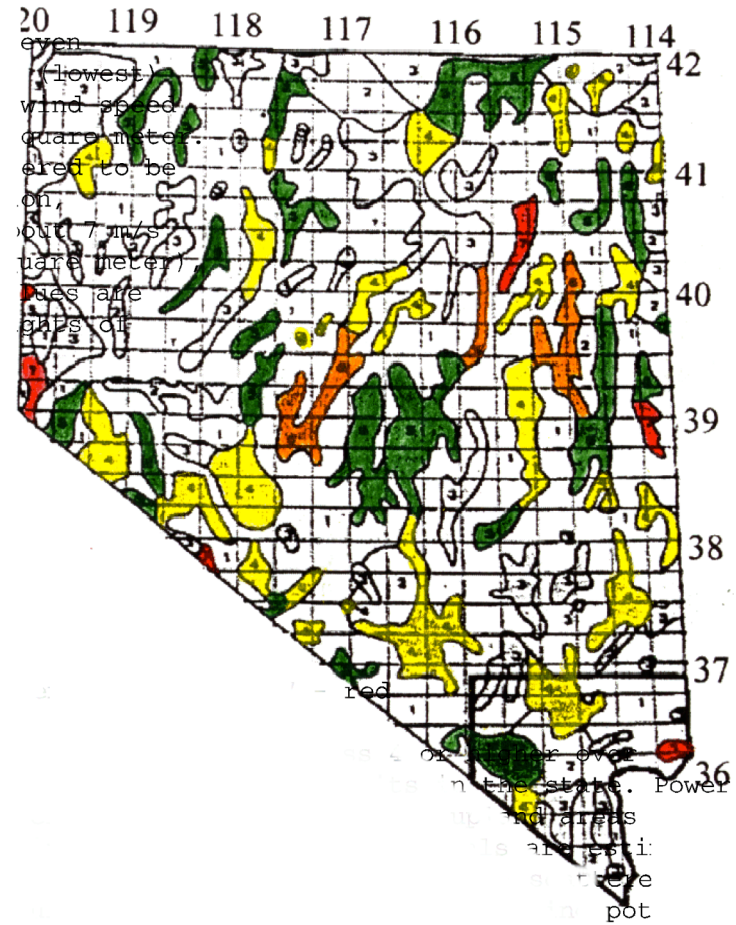
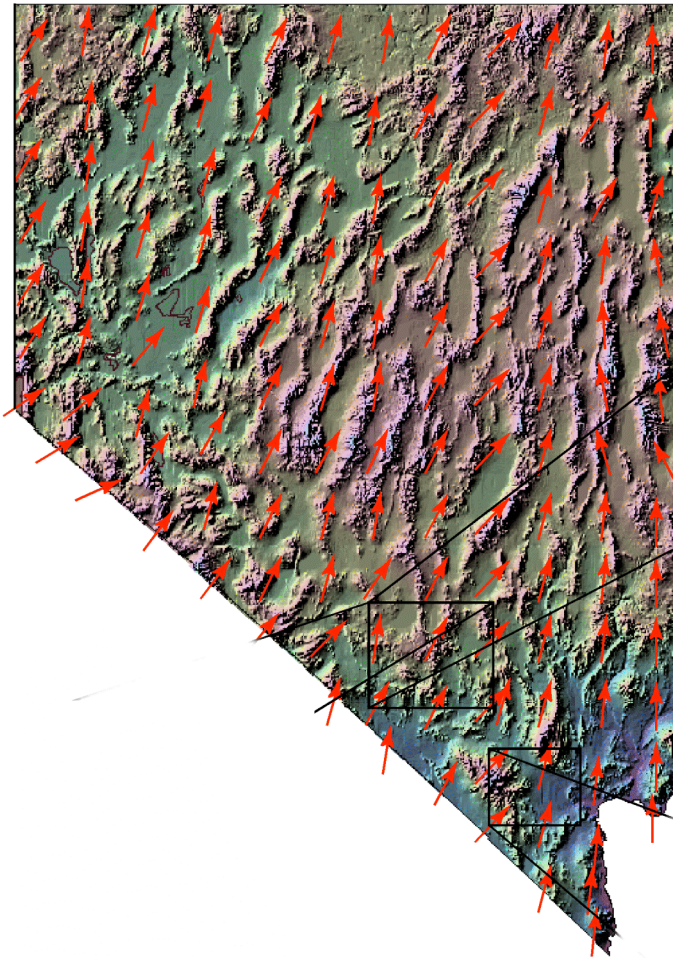
$$\phi(r_j) = \sqrt{r_j^2 + c^2} = \sqrt{(x - x_j)^2 + (y - y_j)^2 + c^2}$$

# Meshless method – con't

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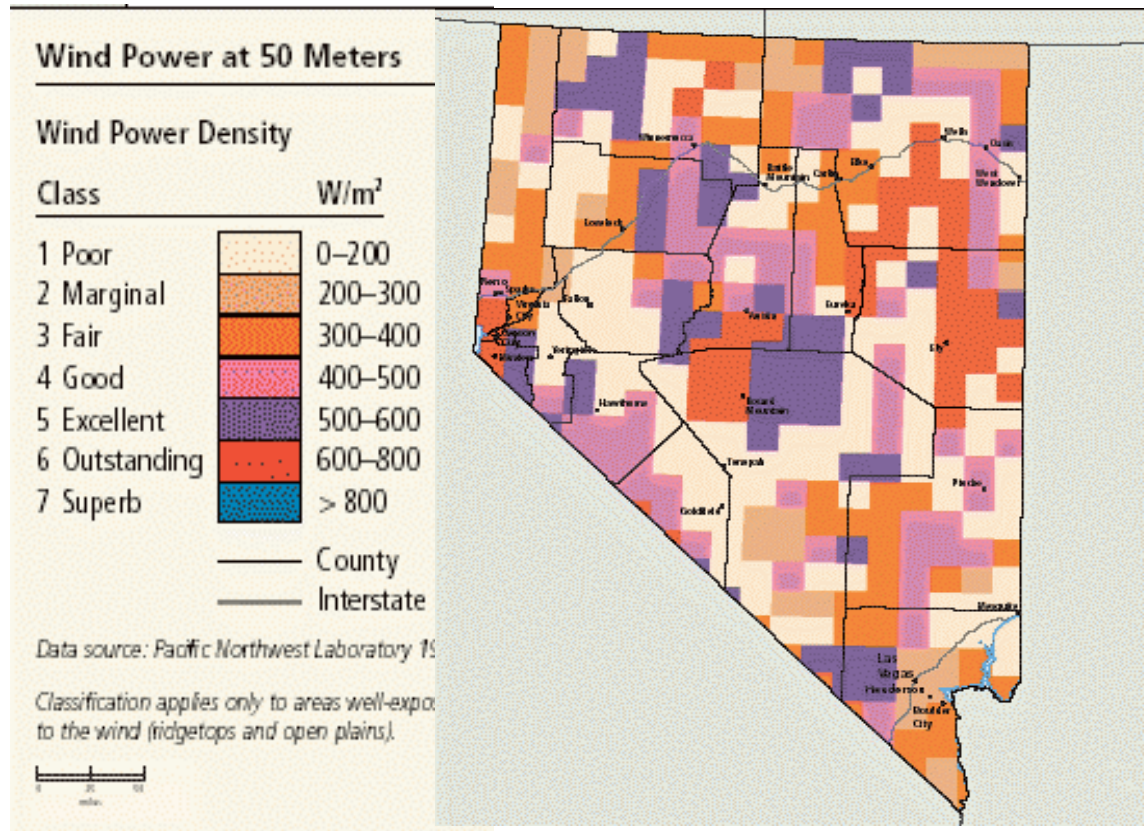
- ◆ Derivatives can be written as

# Wind Potential in Nevada





# Wind Energy



## Notice:

1. Southwestern U.S. as an area of untapped renewable energy resources

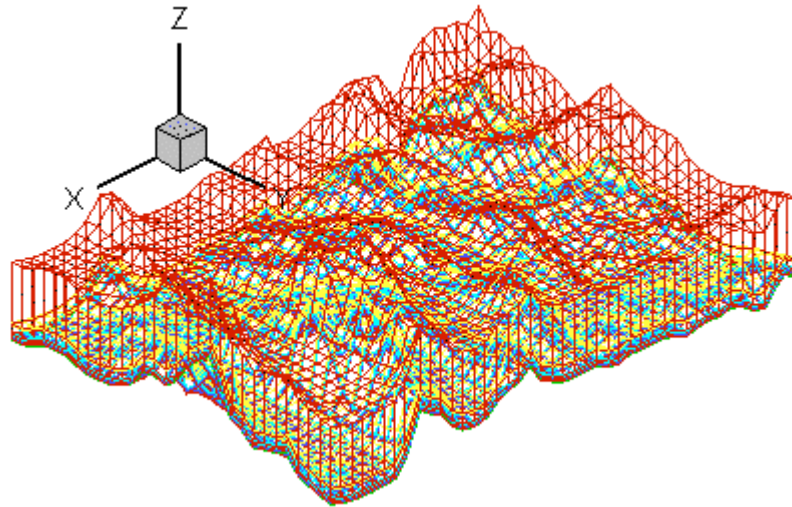
2. Application of numerical models to develop of an annual wind power density map.

## Wind Energy Assessment for Nevada

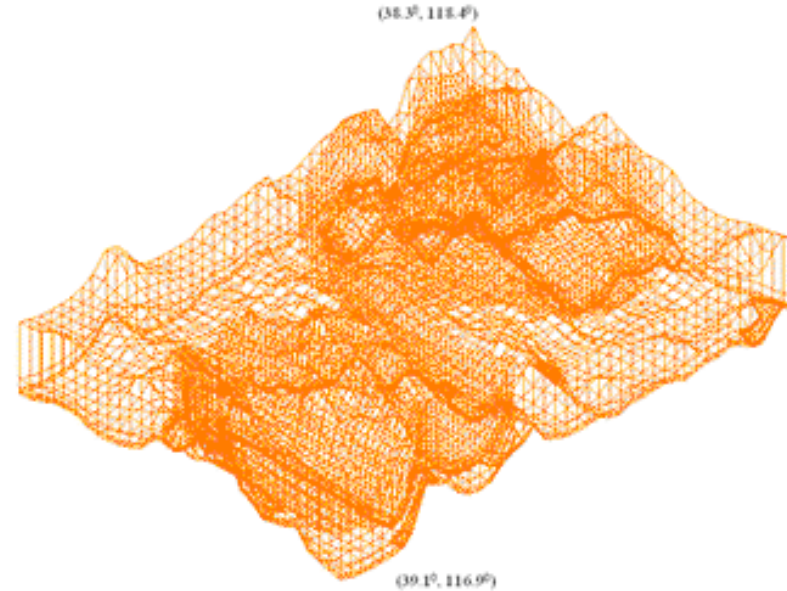
\*Picture sources: [www.awea.org](http://www.awea.org)

# Wind Energy – con't

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**3-D view initial mesh**

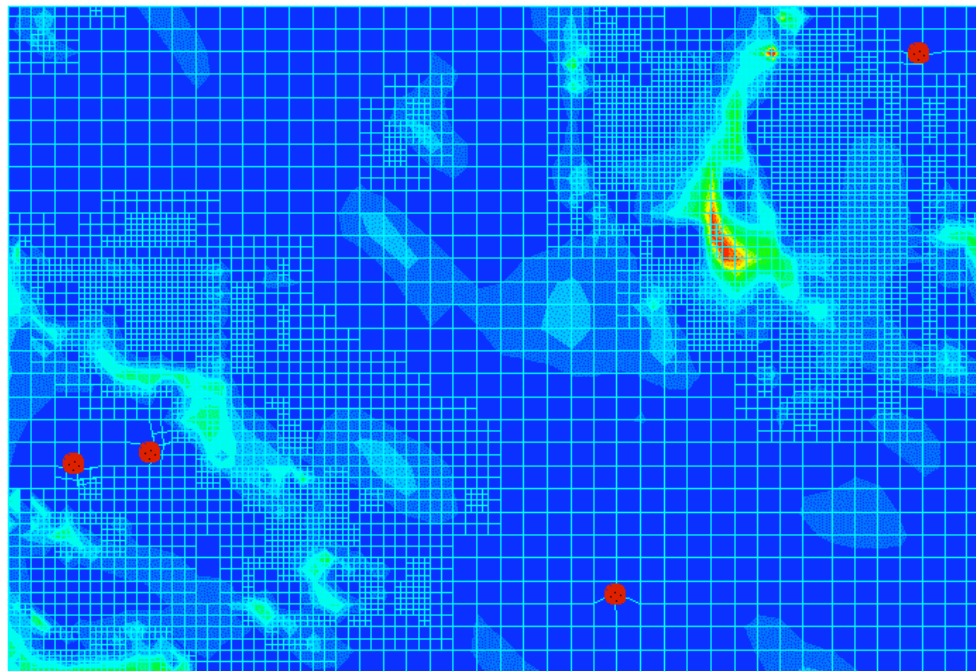
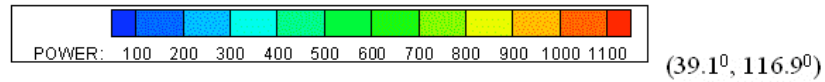


**3-D view *h*-adaptive mesh**

**Mesh for central Nevada Region**

# Wind Energy – Cont.

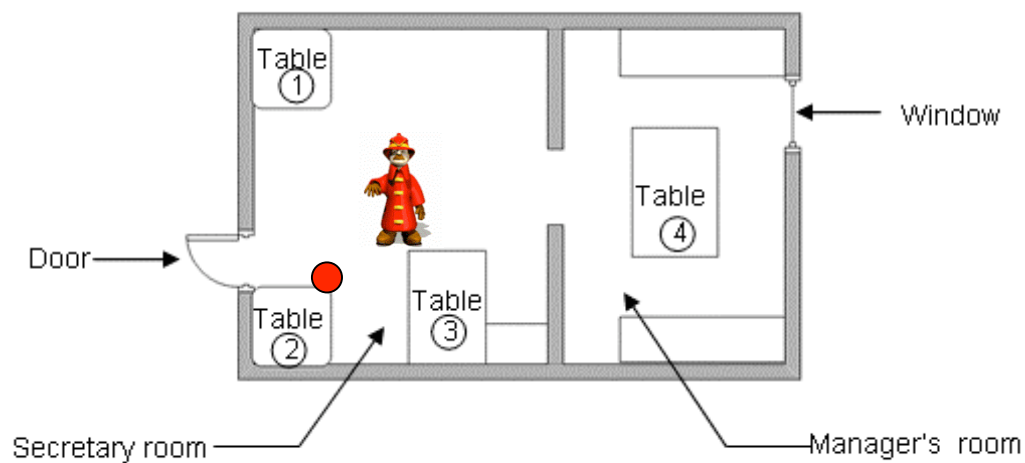
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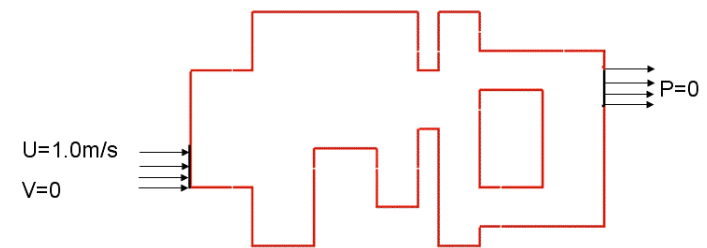
(38.3<sup>0</sup>, 118.4<sup>0</sup>)

Power Density ( $\text{w/m}^2$ ) for 100m layer (December, 2001 data) with  $h$ -adaptation

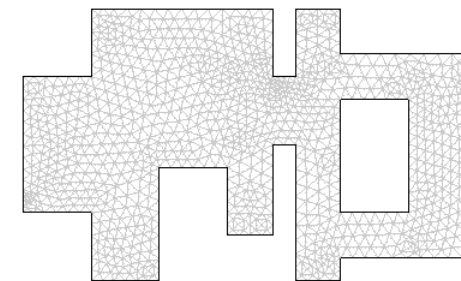
# IAQ simulation for office complex



(a) Problem configuration

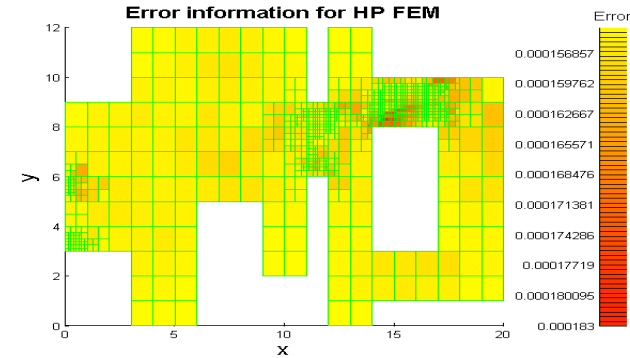
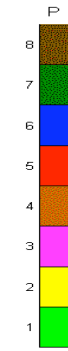
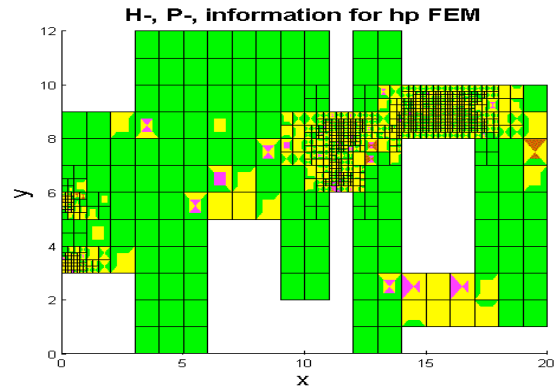


(b) B. C. setting



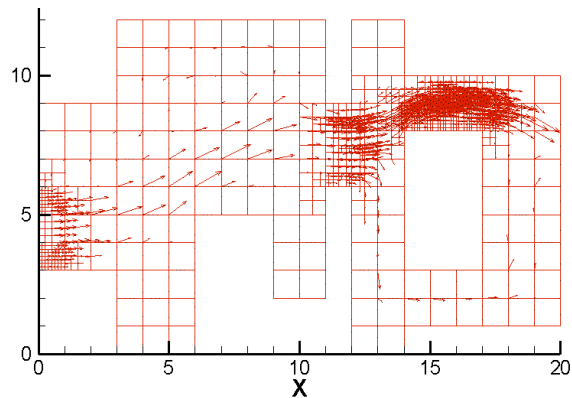
(c) Mesh  
(1932 elements and 9199 DOF)

# IAQ – Cont.

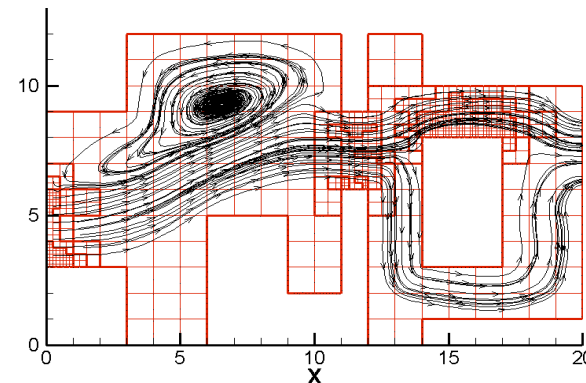


***hp*-adaptive meshes**

**Error distributions**



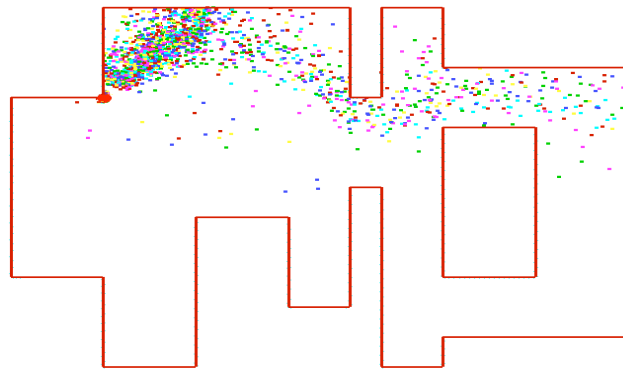
**Velocity Vectors**



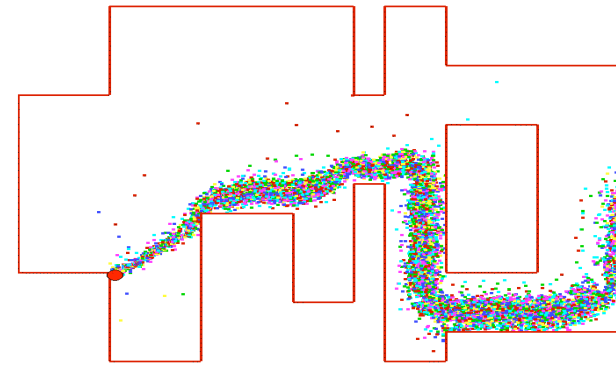
**Streamlines**

# IAQ – Cont.

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(a) Contaminant source in upper table



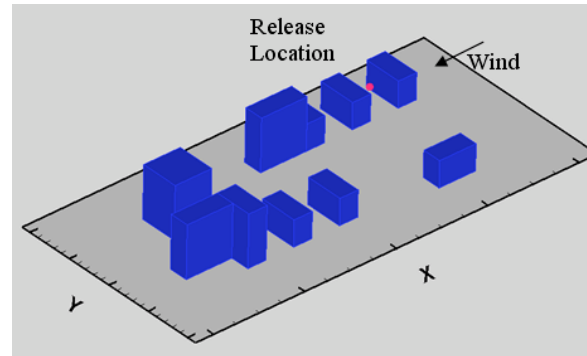
(b) Contaminant source in lower table

**Adaptive results coupled with LPT to simulate  
contaminant dispersion**

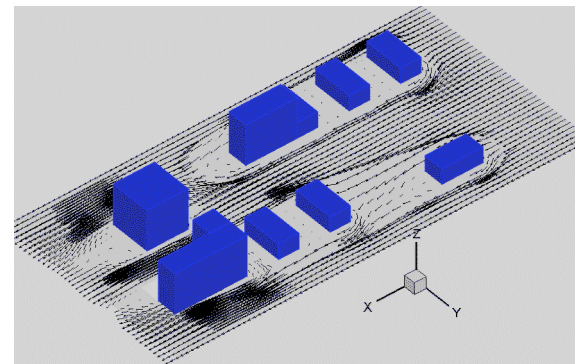
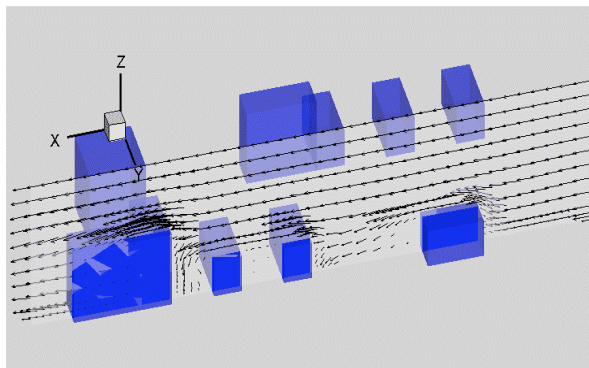
**Notice:** Pollutant is transported and diffused by the ventilation pattern affecting the office complex. Source location is particularly important as the pollutant can travel to either side of the manager's desk within the inner office.

# Contaminant dispersion around buildings

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Buildings layout and flow direction

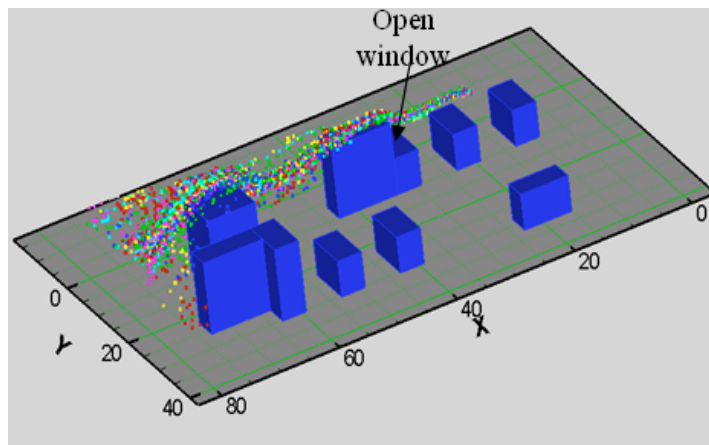


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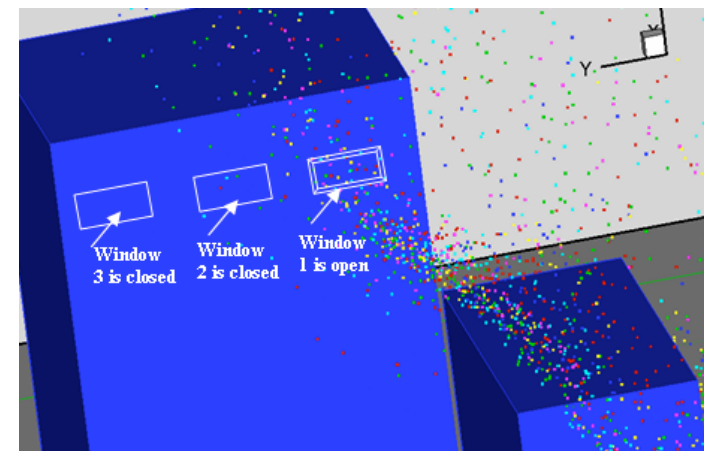
Velocity vectors in vertical and horizontal slices

# Contaminant dispersion around buildings –cont.

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Contaminant dispersion traces around buildings

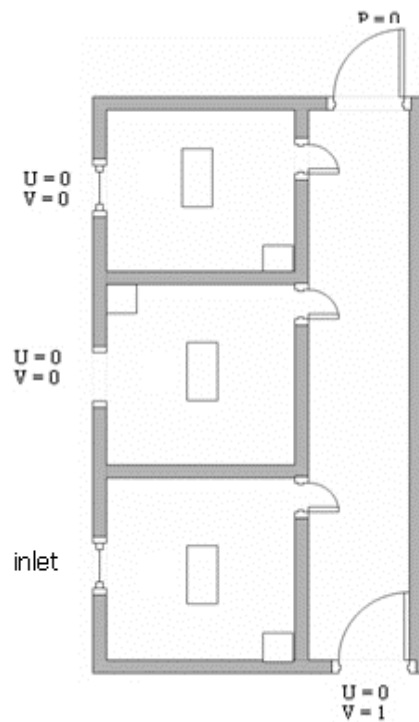


Close view of contaminant entering the first window

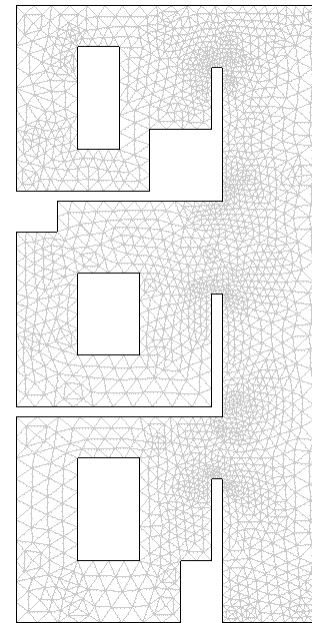


# Simulation results

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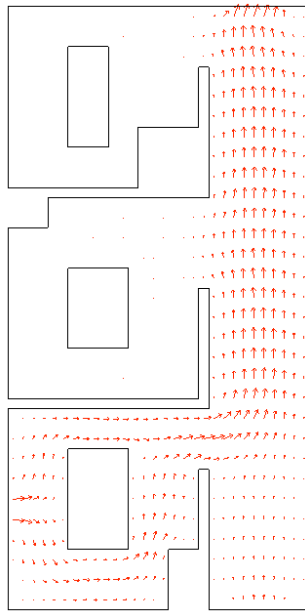
Boundary setting



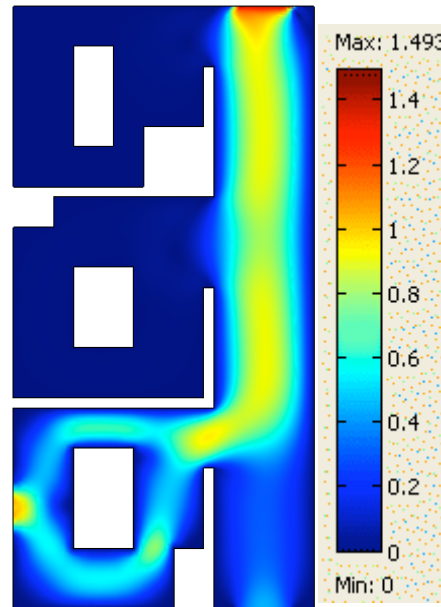
Mesh (3748 elements and 17645 DOF)

# Simulation results – cont.

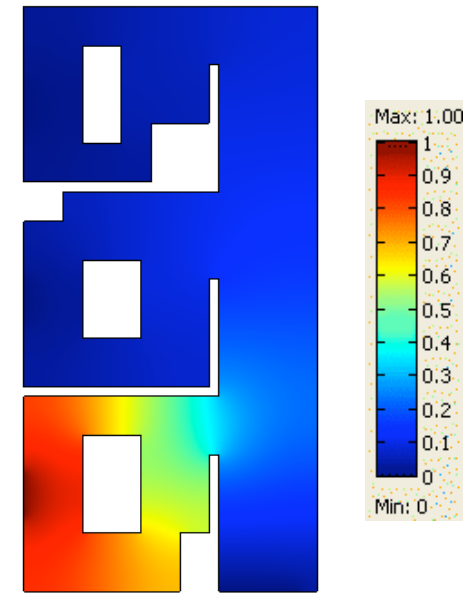
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Velocity vectors



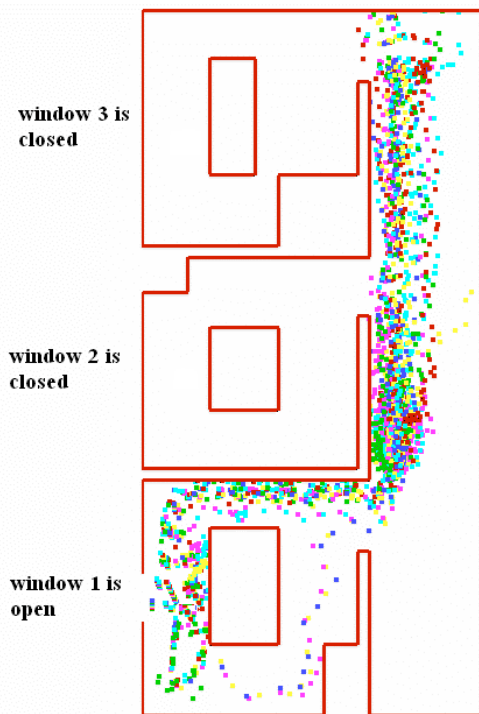
Velocity contours



Species concentration

# Dispersion within building interiors (coupled with LPT)

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Particle dispersion traces

Notice:

1. the contaminant material enters room 1 and becomes dispersed around the desk. Rooms 2 and 3 are essentially contaminant free regions (since both windows are closed) with the air in offices 2 and 3 being relatively stagnant.
2. Because interior air enters in through door 4, the lower part of the hallway is contaminant free.

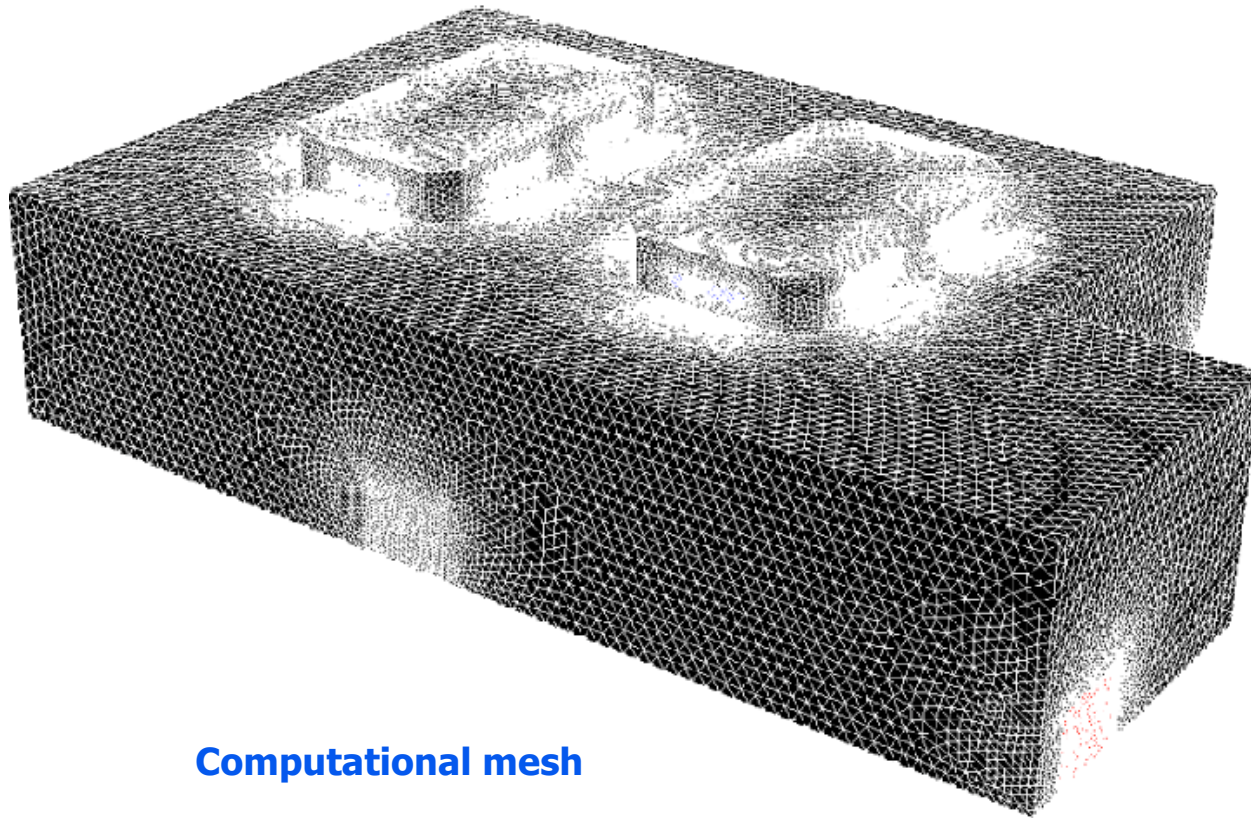
# Marriot Hotel

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# IAQ simulation for JW Marriot Hotel

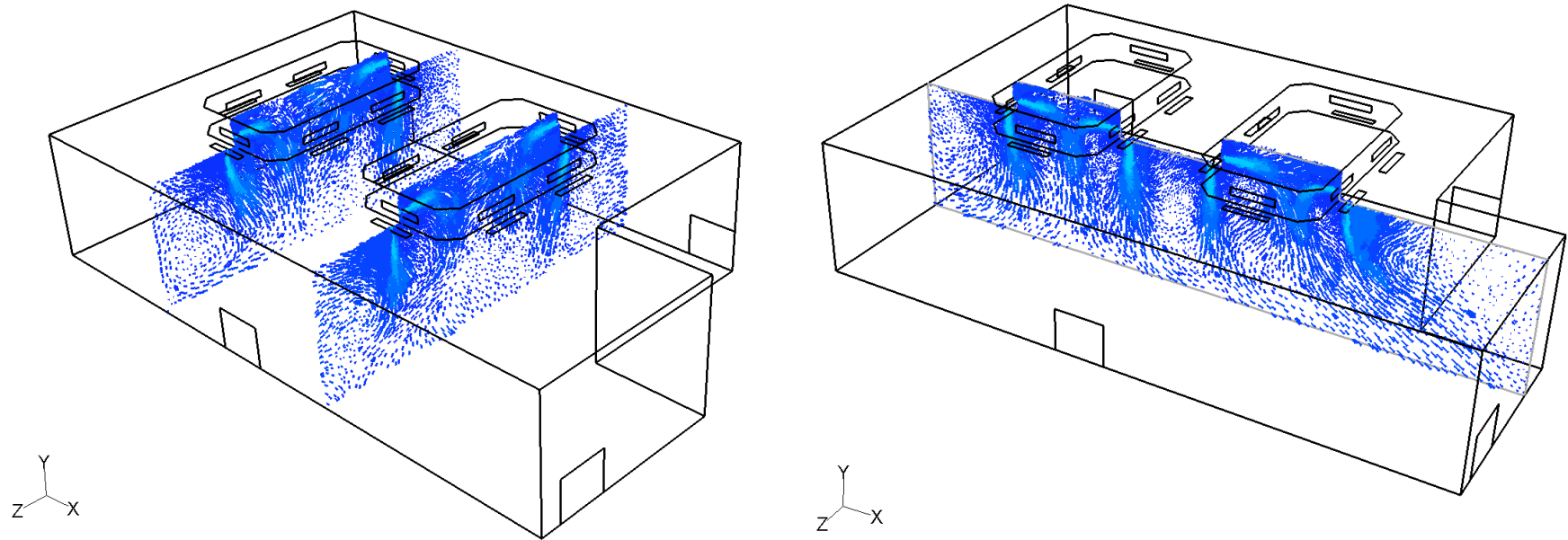
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Computational mesh

# IAQ simulation for JW Marriot hotel – cont.

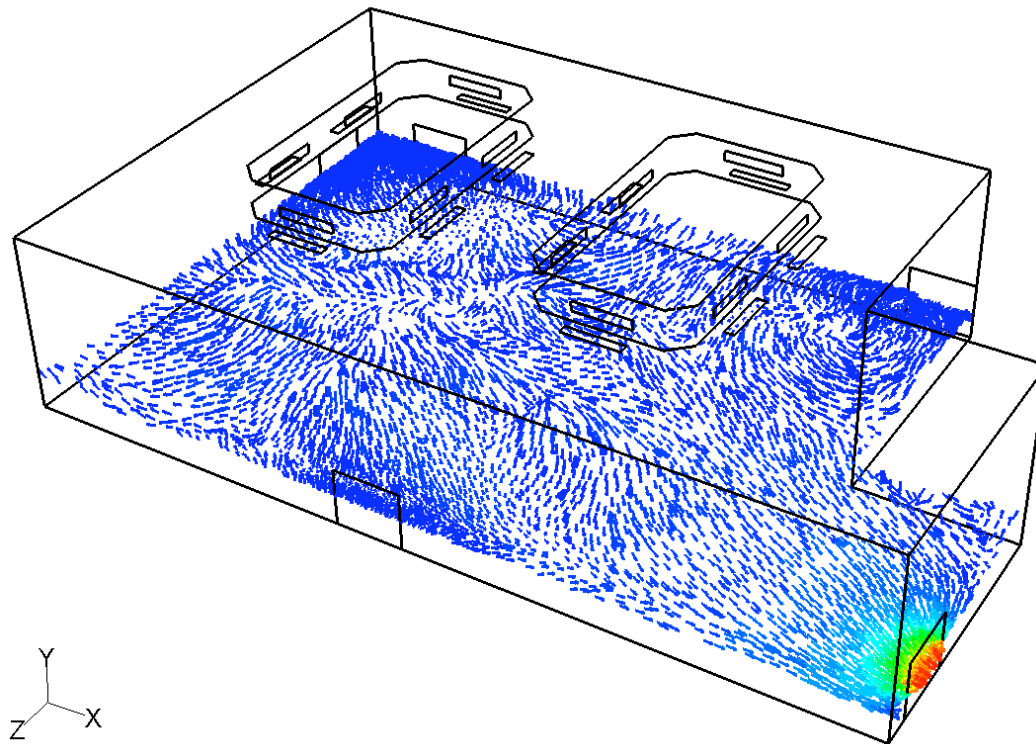
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Velocity vectors at vertical slices

# IAQ simulation for JW Marriot hotel – cont.

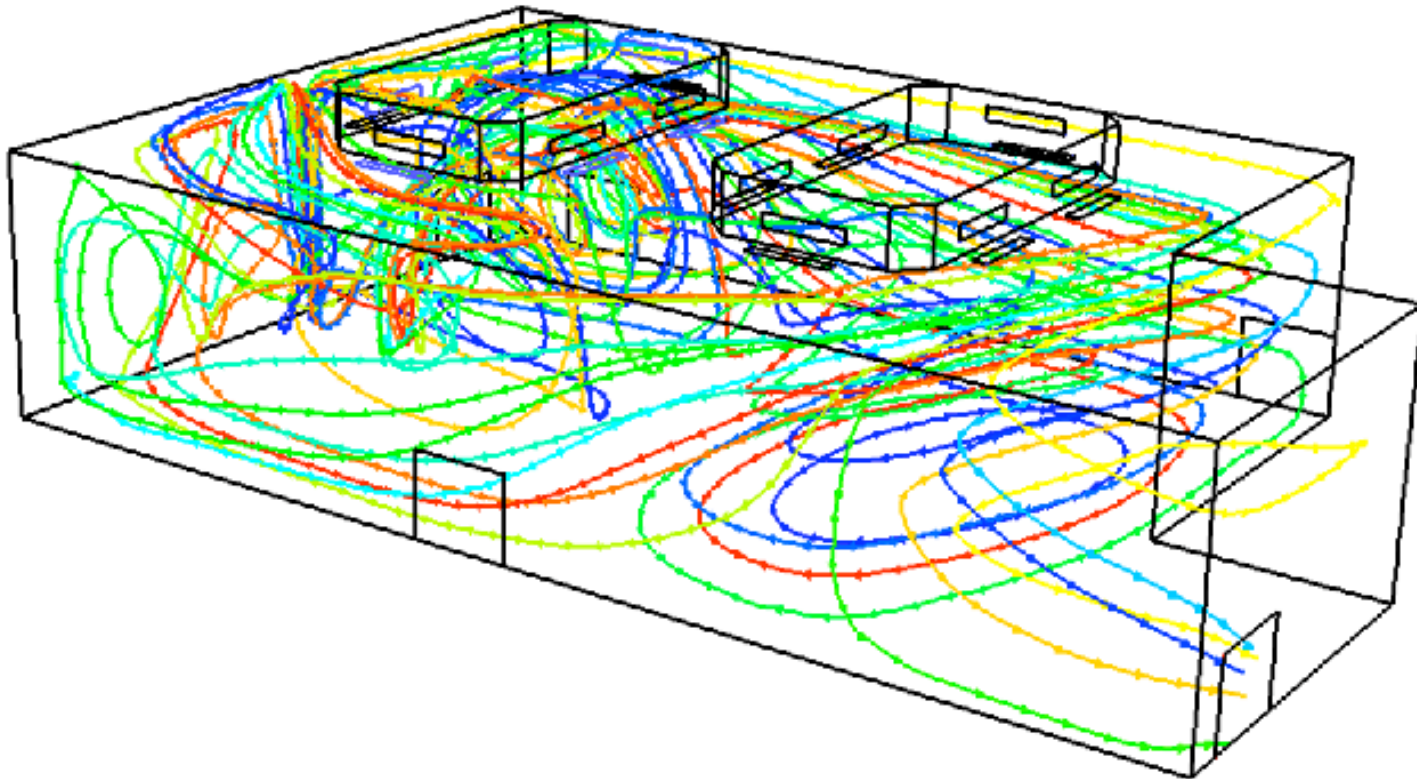
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Velocity vectors at horizontal slice

# IAQ simulation for JW Marriot hotel – cont.

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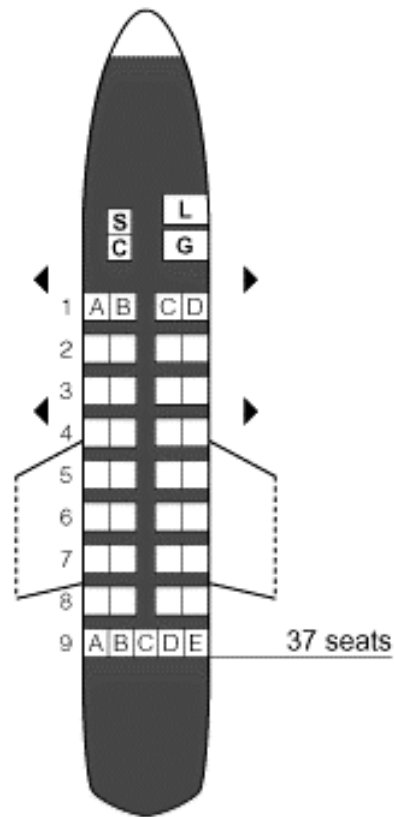


Particle pathlines

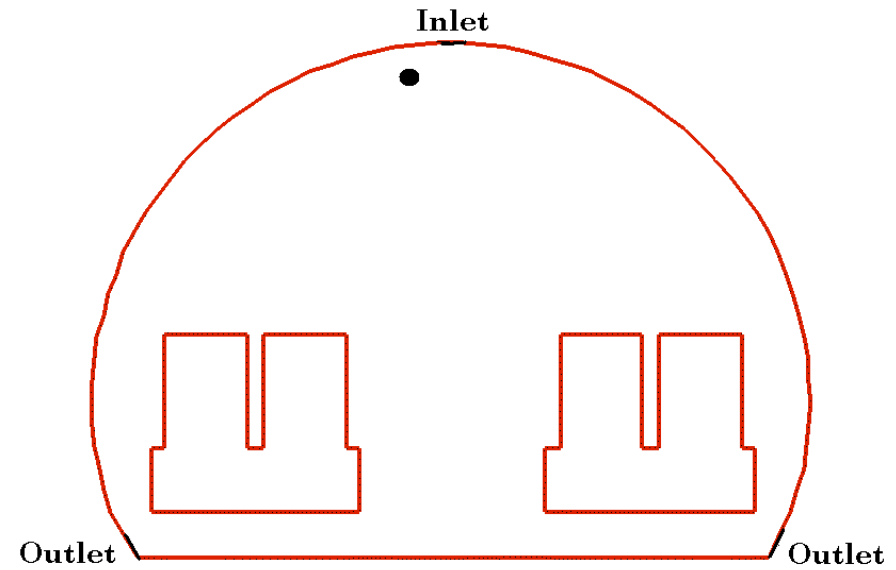
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# Aircraft cabin dispersion



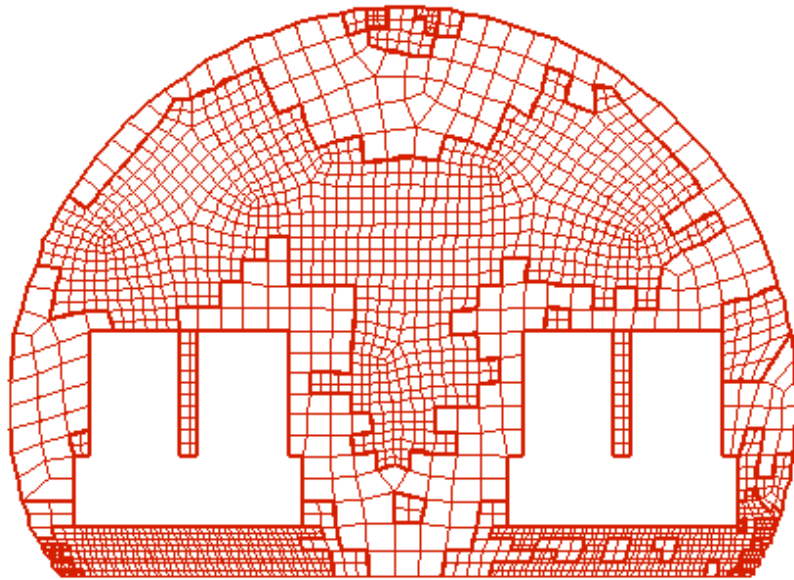
(a) Sample cabin seating map



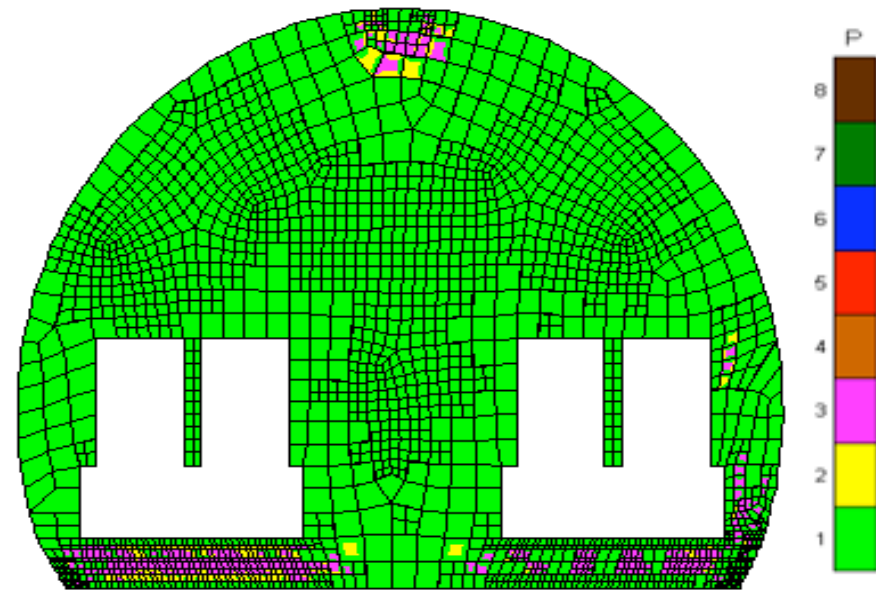
(b) Cross section configuration

# Aircraft cabin – con't

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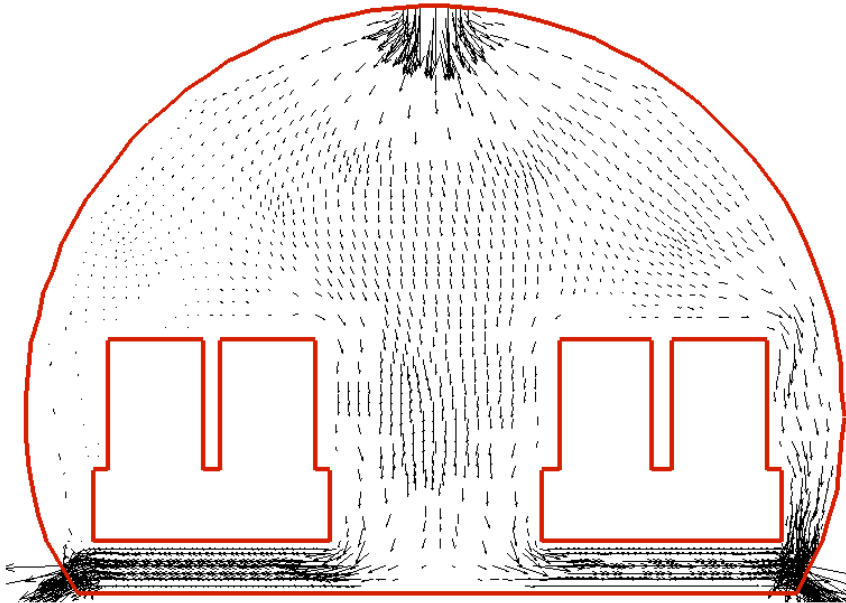
(a) Intermediate  $h$ -adaptive mesh



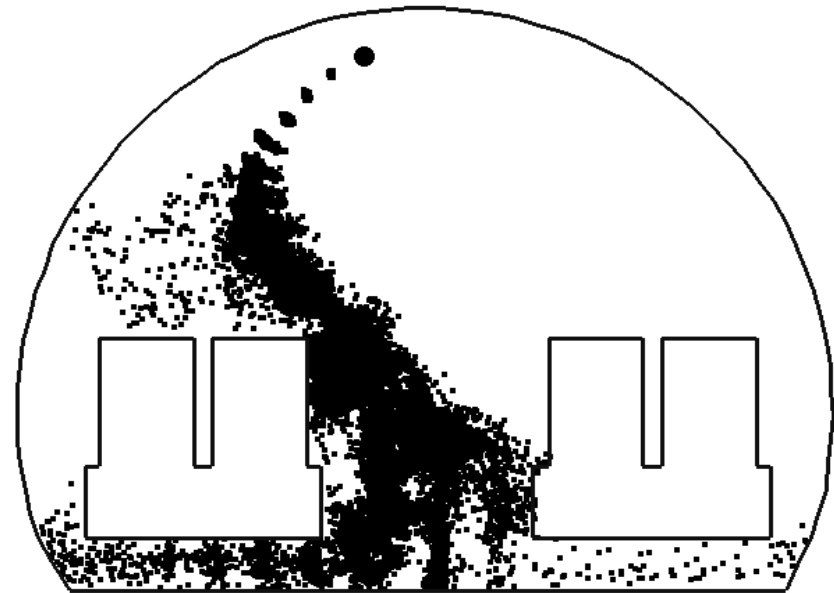
(b) Final  $hp$ -adaptive mesh

# Aircraft cabin – con't

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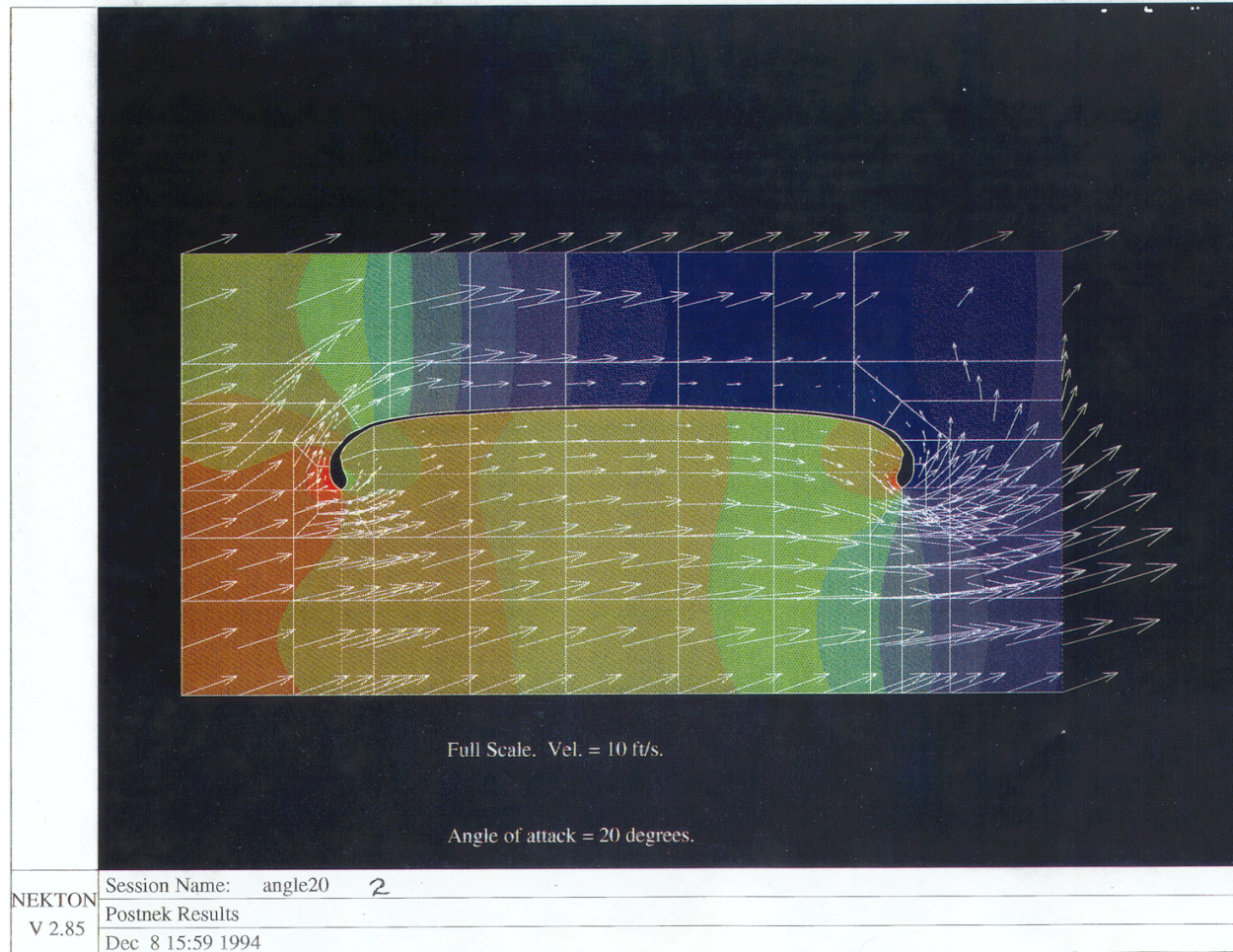


(a) Velocity vectors



b) Contaminant dispersion traces

# World's largest Frisbee



# World's largest Frisbee

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Also set unofficial distance record –over 1200 feet

# Solar-powered airplane











# Solar water pump – 5000 ft depth

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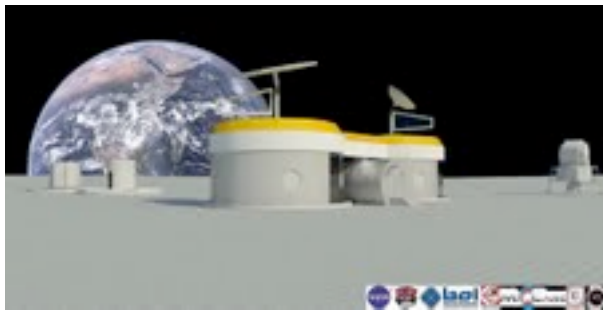
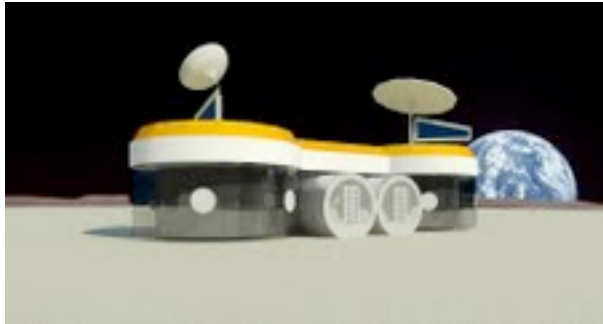
# Lunar-Mars Habitats

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- ◆ **Grant from NASA to design and build prototype Lunar and Mars habitats**
    - ✧ Lunar surface – regolith – will compress and form walls without need of water
    - ✧ Mars surface – will make bricks to form vaults and domes
    - ✧ Built Mars habitat in 2002 using foam panels an 16-gauge steel
    - ✧ Mars habitat obtained by Mars Society (Zubrin)
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# Lunar Habitat

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# Concluding Remarks

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- ◆ **Multiphysics – COMSOL**
- ◆ **Multiscale – not so easy**
- ◆ **Inclusion of stochastic/inverse techniques**
- ◆ **3-D imaging**
- ◆ **Too much data – how to make sense of it**
- ◆ **Matlab/Maple/Mathematica – where's Fortran?**
- ◆ **Advances in meshless methods – getting away from mesh generation**
- ◆ **Faster computers – better algorithms?**

# Contacts

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[www.ncacm.unlv.edu](http://www.ncacm.unlv.edu)

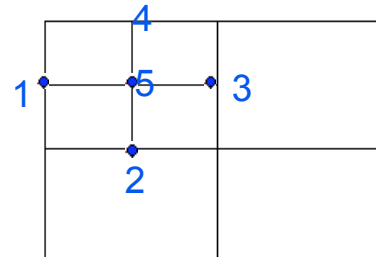
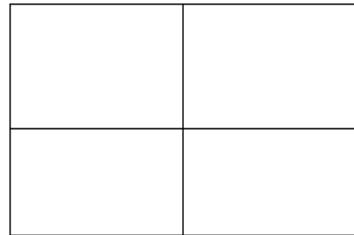
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# Questions?

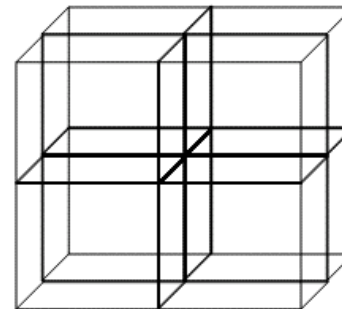
# Element rules for h-adaptation

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- ◆ A refined quadrilateral element creates 4 children and 5 new vertex nodes\*.



- A refined hexahedral element creates 8 children and 19 new vertex nodes.



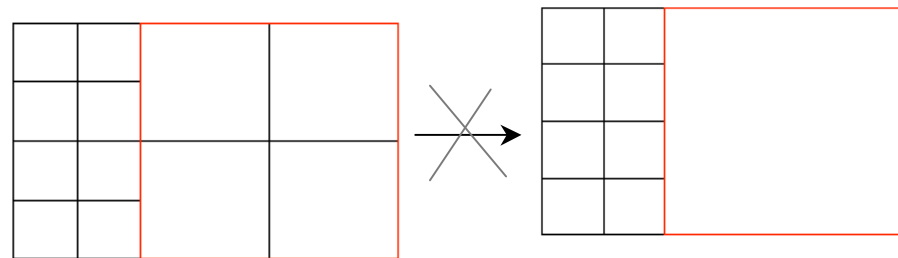
Note: only vertex nodes are showed here higher order nodes follow the same rule



# Element rules – Cont.

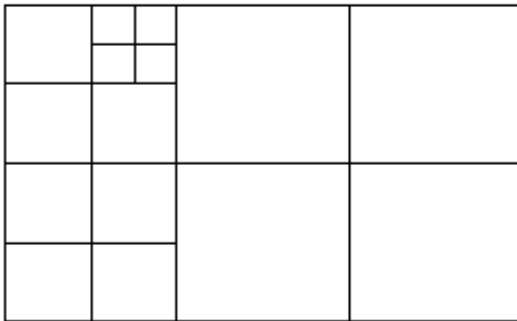
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- ◆ An element may be recovered only if its neighbors are at the same or less level

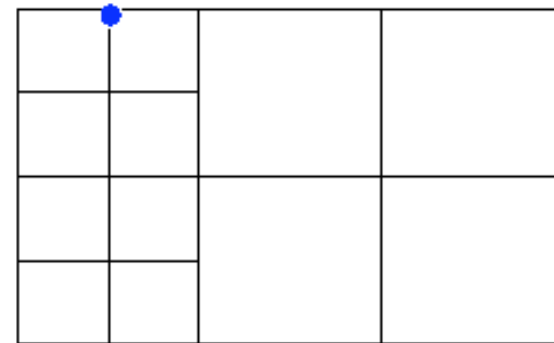


# Nodal rules

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Wrong !



- ◆ **1-Irregular mesh rule must be followed**

- A vertex node along a boundary is not a hanging vertex node.