OUT-OF-EQILIBRIUM STATISTICAL AND DYNAMICAL PROPERTIES OF LONG-RANGE INTERACTING SYSTEMS

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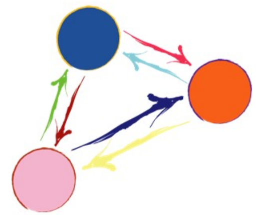
Doctoral Study Programme in Physics

Interactions

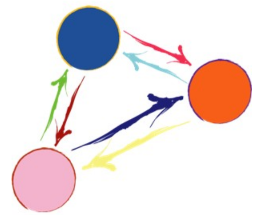


Short-range

Long-range

Interactions

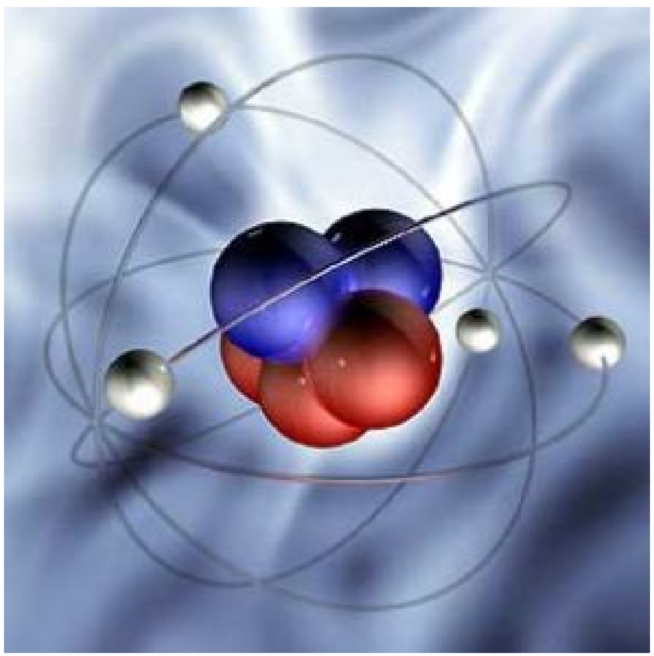
Atomic nucleus with radius of action is only 10^{-12} - 10^{-13} centimeter



Gravitational systems, interaction betweeen charges particles or dipoles

Short-range

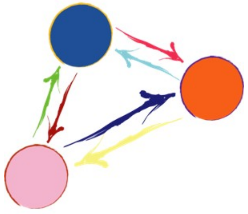
Long-range



$$V(r) = \frac{C}{r^{\alpha}}$$ - two-body interaction potential

r – scalar distance between the two elements,
d – dimension of the system.

Interactions



Short-range

$$\alpha > d$$

Long-range

$$\alpha \leq d$$

Why should we study these systems?

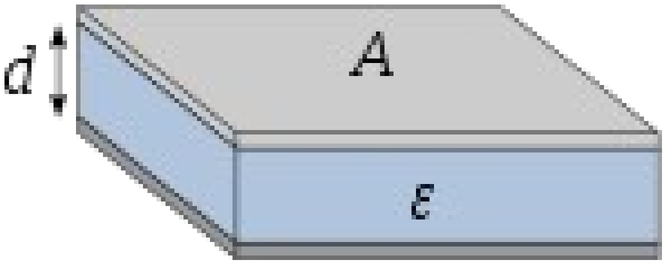
Because of their peculiar phenomena:

-Quasi-stationary states (QSS)
-Non-additivity of energy
-Negative specific heat

The Hamiltonian Mean Field (HMF) model

$$V(r) = \frac{C}{r^{\alpha}}$$

Mean-field model: $\alpha = 0$



Electric field inside capacitor:

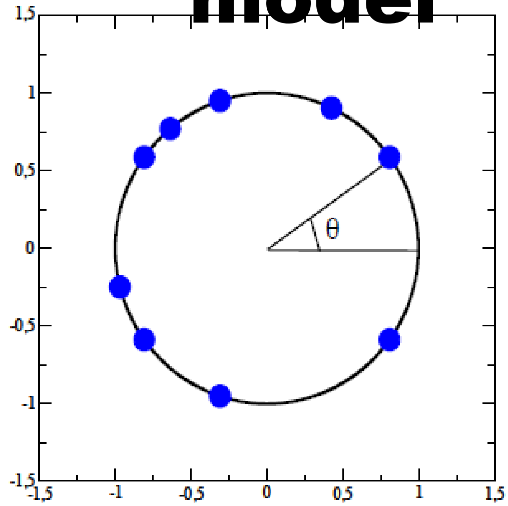
$$E = \frac{Q}{\varepsilon_0 \varepsilon_{\mathrm{r}} A}$$

The force acting on a particle with charge *q* placed inside capacitor:

$F = qE$

(the force does not depend on the place of the particle location)

The Hamiltonian Mean Field (HMF) model

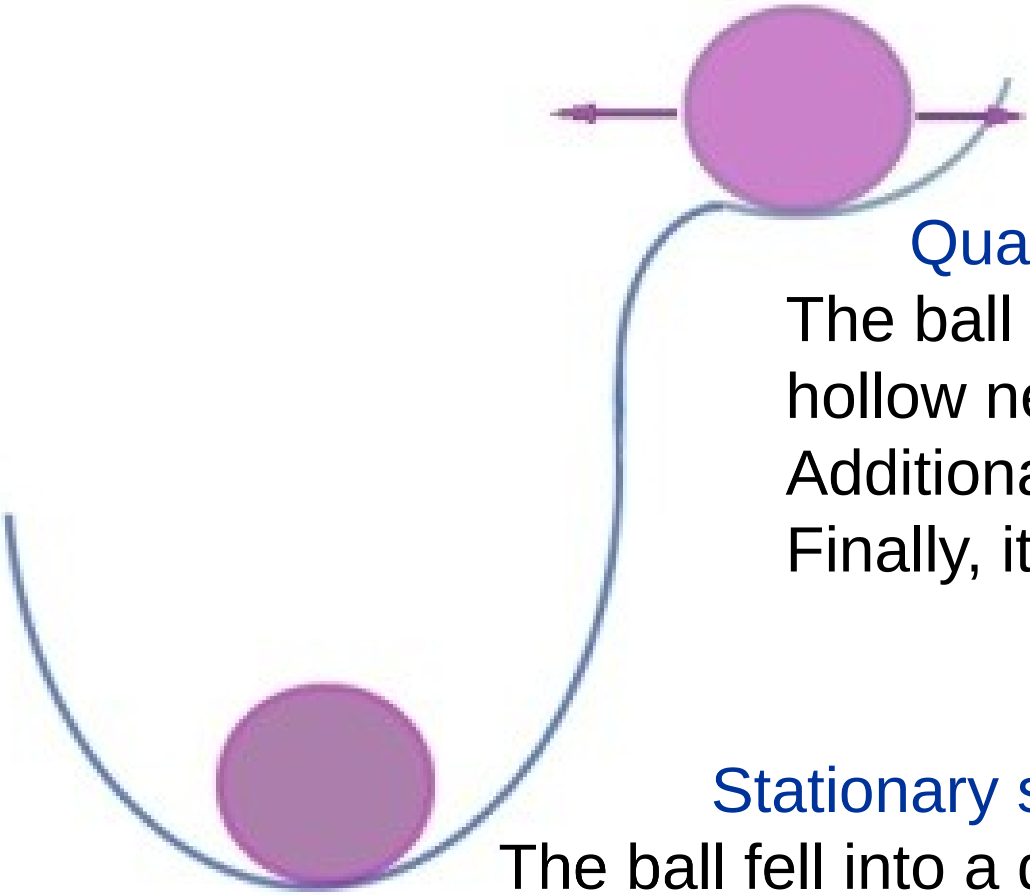


Hamiltonian: $$H = \frac{1}{2}\sum_{j=1}^{N} p_j^2 + \frac{\epsilon}{2N}\sum_{i,j=1}^{N}[1-\cos(\theta_i-\theta_j)]$$

θ_j – particle position (angle); p_j – momentum.

Equations of motion:
$$\dot{\theta}_i = p_i$$
$$\dot{p}_i = -\frac{\epsilon}{N}\sum_{i,j=1}^{N}\sin(\theta_i-\theta_j).$$

Stationary and quasi-stationary states



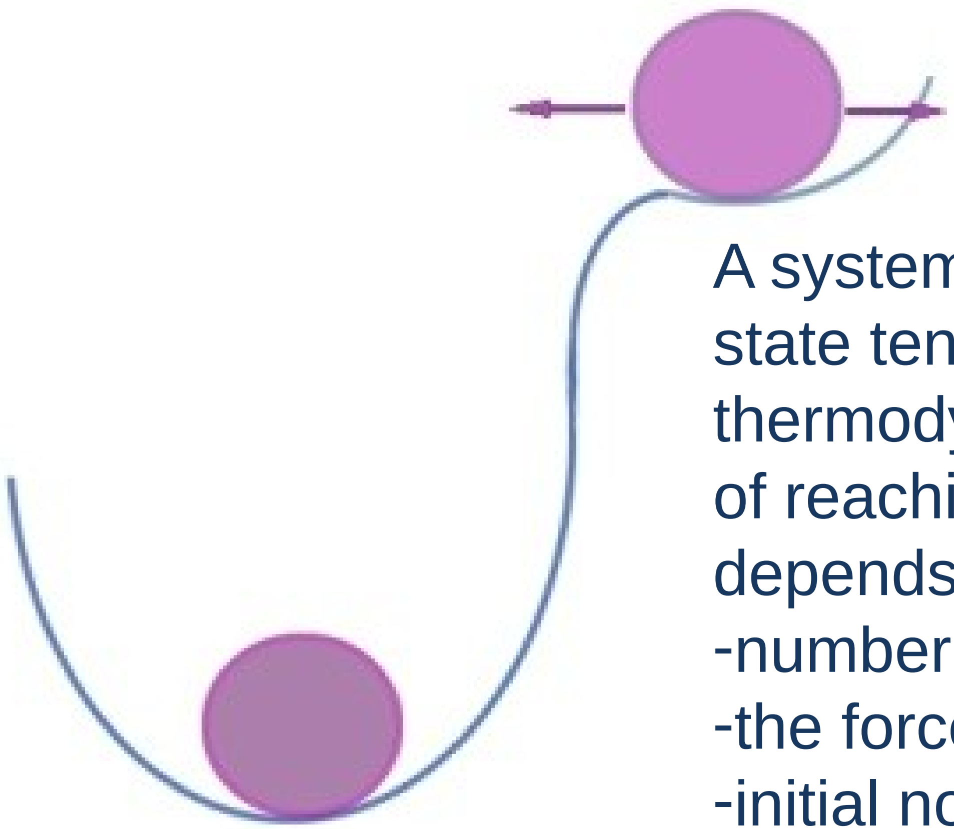
Quasi-stationary state:

The ball is located in a shallow hollow near the pit. Additionally, it fluctuates. Finally, it will fall into the pit.

Stationary state:

The ball fell into a deep pit. It can stay in this place for a long time

Quasi-stationary states (QSS)



A system in out-of-equilibrium state tends to establishing a thermodynamic equilibrium. Time of reaching an equilibrium state depends on many factors:

-number of particles

-the force between the particles

-initial non-equilibrium state

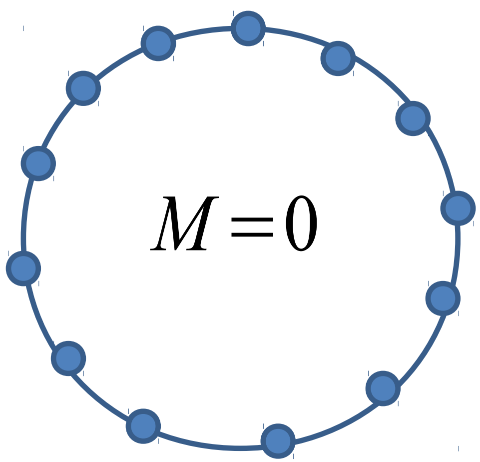
-external action (e.g. external magnetic field).

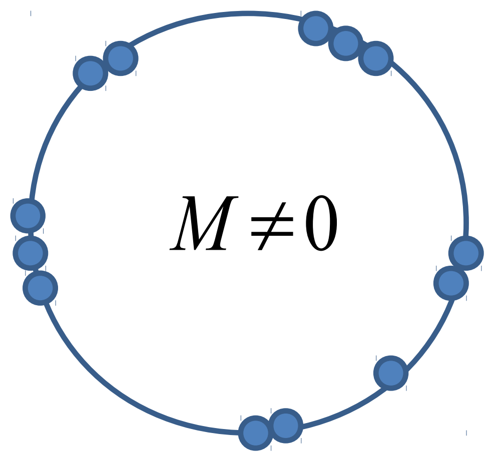
Quasi-stationary states (QSS)

Magnetization

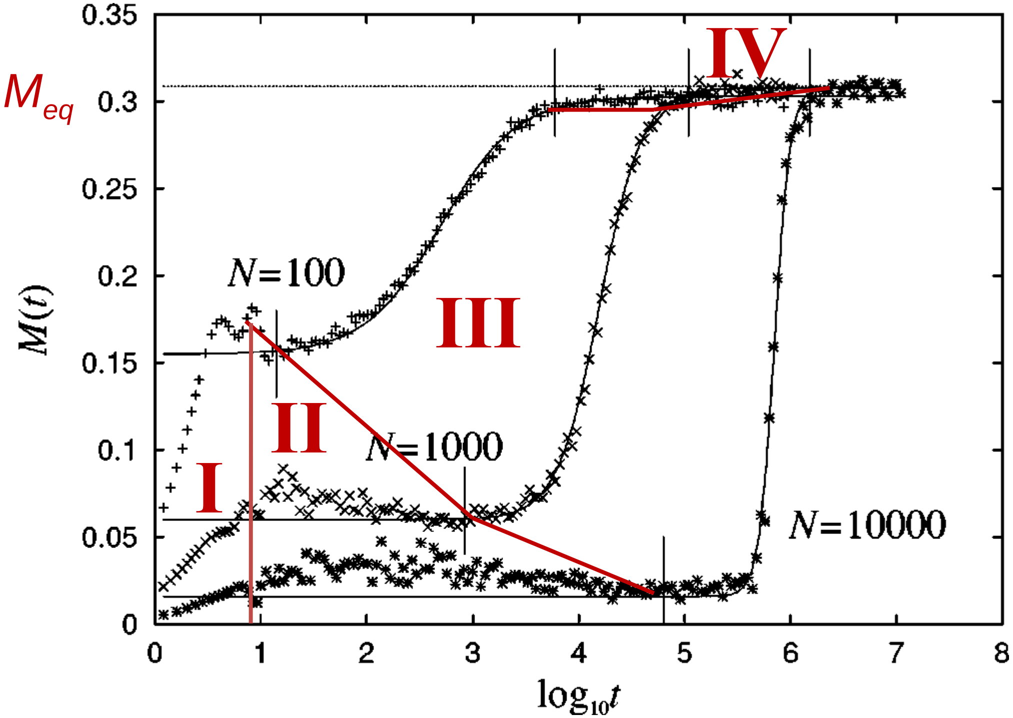
$$M = |\mathbf{M}| = |M_x + iM_y| = |\sum \mathbf{m}_i/N|.$$

$$\mathbf{m}_i = (\cos\theta_i, \sin\theta_i)$$





Quasi-stationary states (QSS)



The time of trapping the system in QSS depends on number of particles N as $t_{QSS} \sim N^{\gamma}, \quad \gamma \approx 1.7$

Three stages of relaxation can be identified:

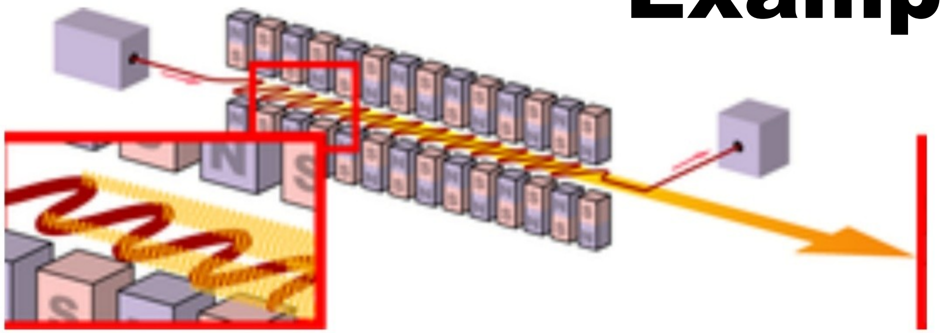
I. Transitory regime (“violent relaxation”)

II. QSS

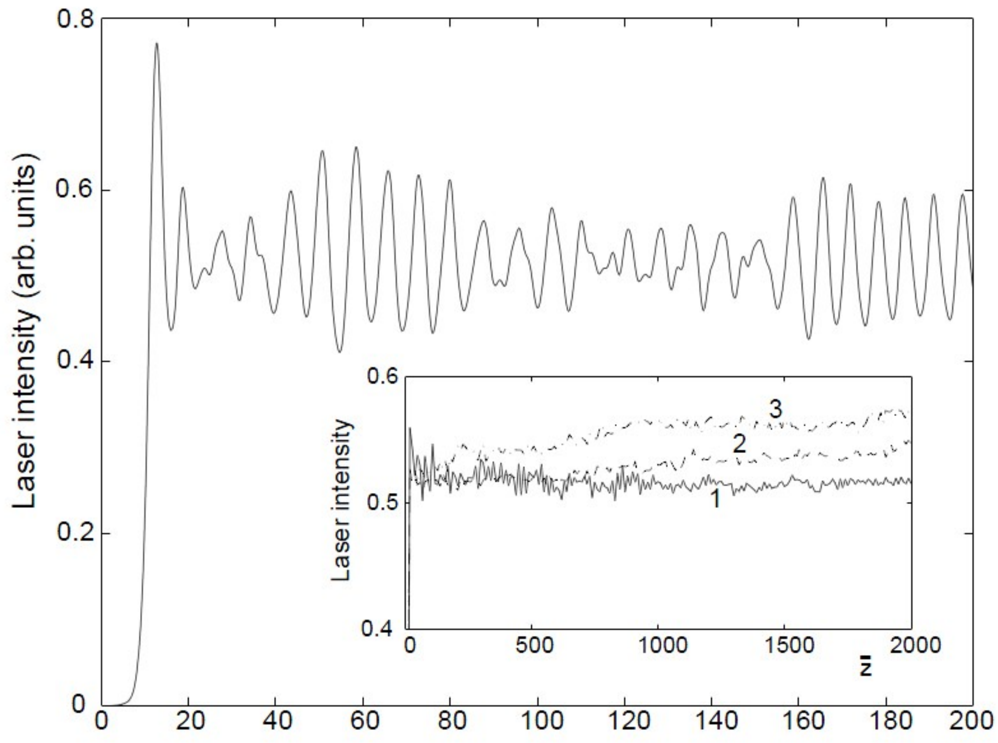
III. Magnetization rapidly increases towards its equilibrium value M_{eq}

IV. Reaching the equilibrium

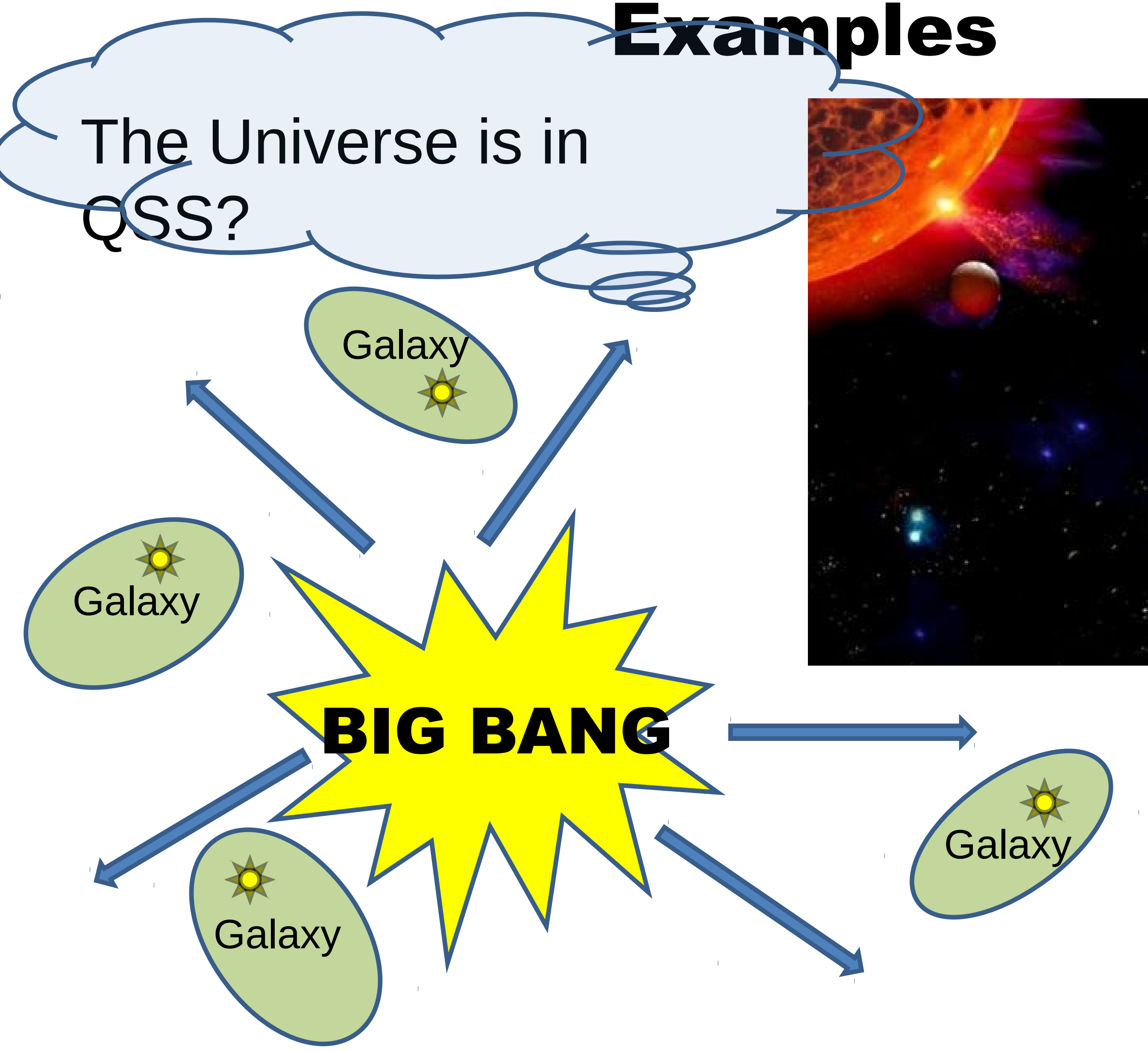
Quasi-stationary states (QSS). Examples



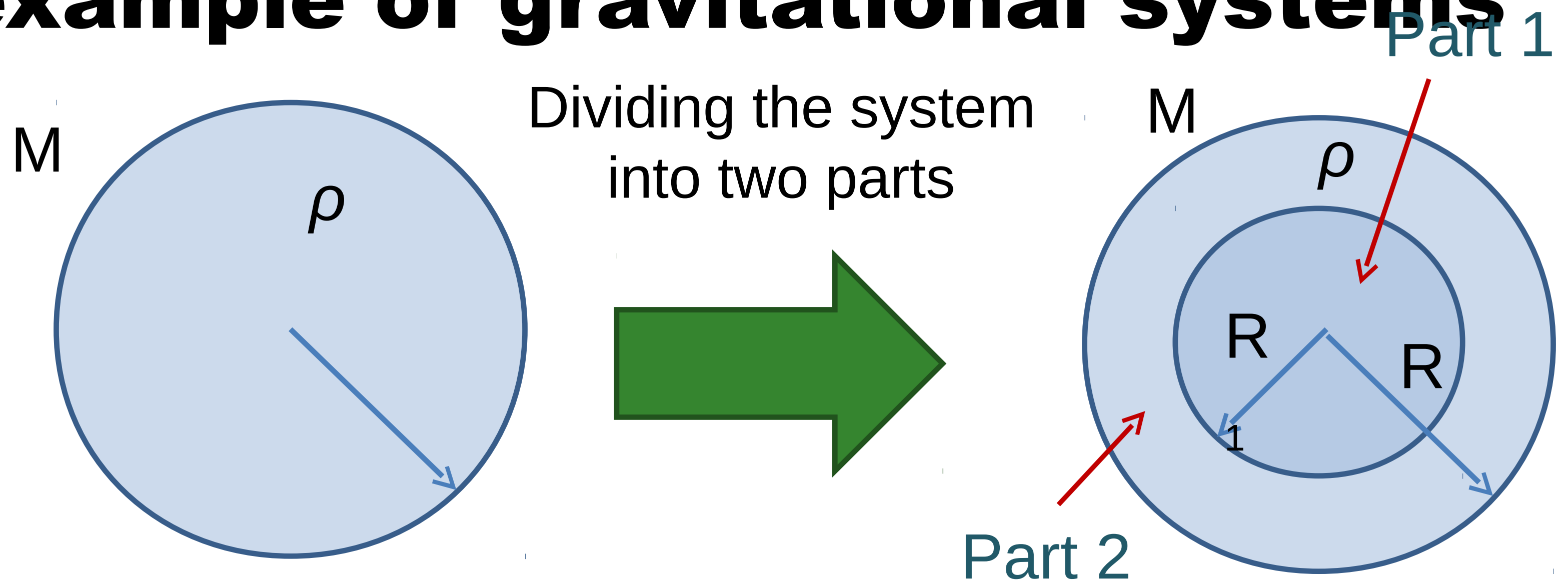
Free-electron laser



Quasi-stationary states (QSS). Examples



Non-additivity of energy by the example of gravitational systems



Uniform distribution of density ρ.
Potential energy is calculated as

$$V_{\text{total}} = \int_0^R \frac{G(\rho \frac{4}{3}\pi r^3)\mathrm{d}m}{r},$$

$$\mathrm{d}m = \rho 4\pi r^2 \mathrm{d}r.$$

$$V_{\text{total}} = G\rho^2 \frac{(4\pi)^2}{3} \frac{R^5}{5}.$$

Uniform distribution of density ρ.
Potential energy is calculated as

$$V_1 = G\rho^2 \frac{(4\pi)^2}{3} \frac{R_1^5}{5}.$$

$$V_2 = G\rho^2 \frac{(4\pi)^2}{3} \left(\frac{R^5}{5} - \frac{R_1^3 R^2}{2} + \frac{3R_1^5}{10} \right)$$

Lynden-Bell approach

$N \to \infty$ ➡ System is described by Vlasov equation:

$$\frac{\partial f}{\partial t} + p\frac{\partial f}{\partial \theta} - \{M_x[f]\sin(\theta) - M_y[f]\cos(\theta)\}\frac{\partial f}{\partial p} = 0$$

$$s(\bar{f}) = -\int \mathrm{d}p\mathrm{d}\theta \left[\frac{\bar{f}}{f_0}\ln\frac{\bar{f}}{f_0} + \left(1 - \frac{\bar{f}}{f_0}\right)\ln\left(1 - \frac{\bar{f}}{f_0}\right)\right]$$ - entropy

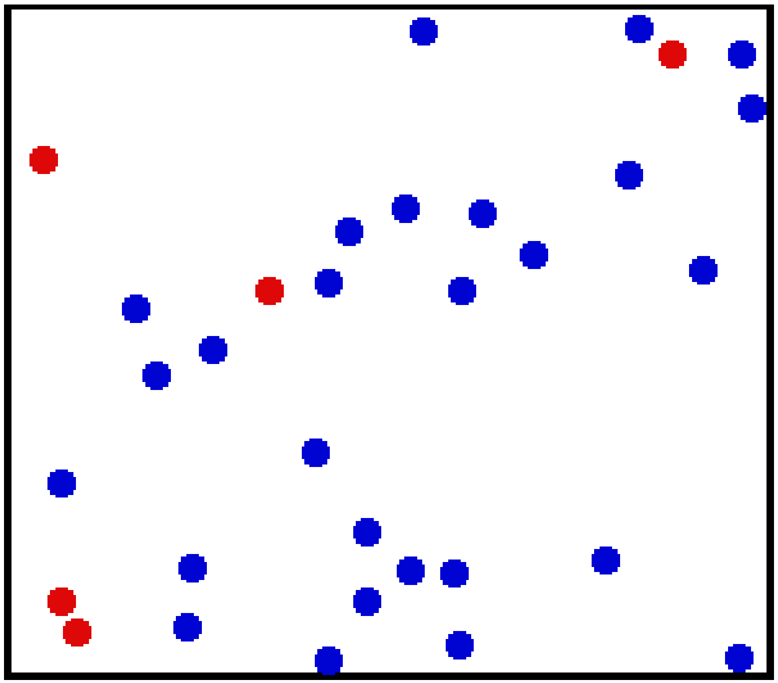
A coarse-grained system

Two approaches

Dynamical

Statistical
(Lynden-Bell)

Thermodynamic and kinetic temperatures



Temperature is a value which characterizes an average kinetic energy of the particles in the system

Kinetic definition:

$$T_{\rm kin} = \int p^2 F(\theta, p) d\theta dp$$

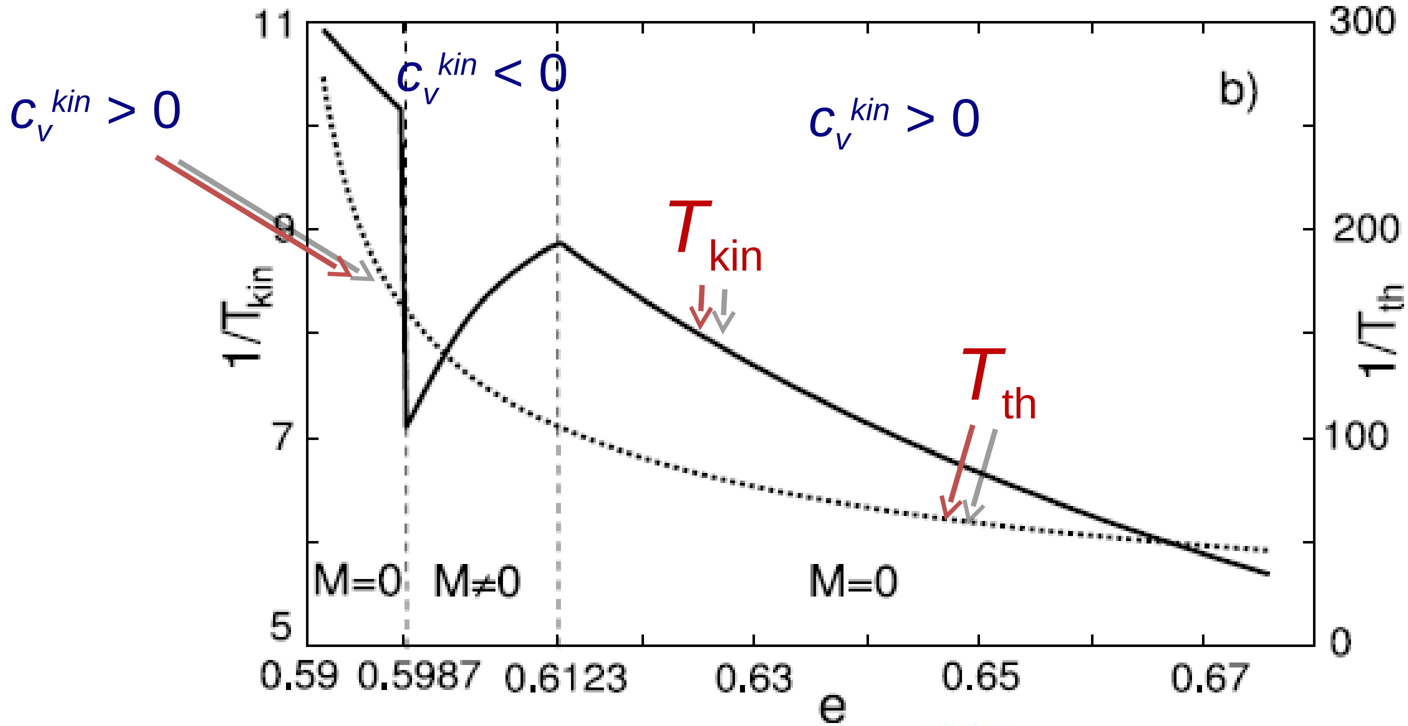
Thermodynamic definition:

$$T_{\rm th} = (\partial s / \partial e)^{-1}$$

In equilibrium state: $T_{kin} = T_{th}$

In QSS often $T_{kin} \neq T_{th}$

Inequality of thermodynamic and kinetic



Specific heat: $c_v^{\text{mic}} = \frac{\partial e}{\partial T}$

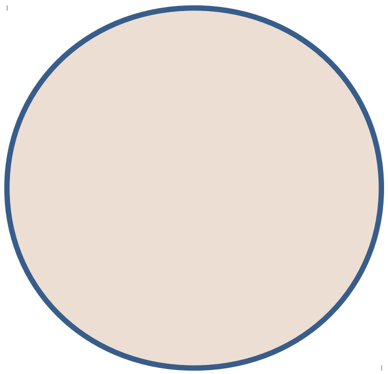
For QSS, a thermodynamic temperature is a monotonous function of the energy of the system and $c_v^{kin} > 0$

while the kinetic temperature experiences a jump in a magnetization region and, hence, $c_v^{kin} < 0$ in this region

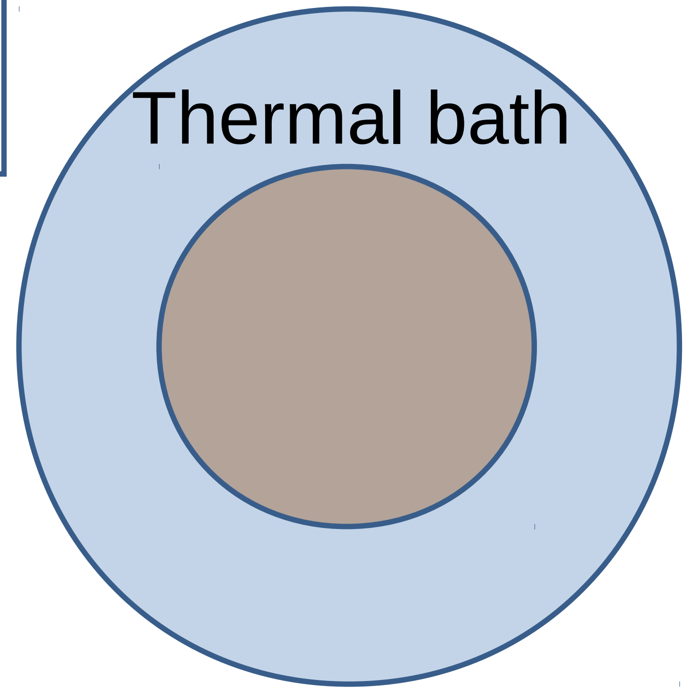
Canonical ensemble

HMF

$$H = \frac{1}{2}\sum_{j=1}^{N} p_j^2 + \frac{\epsilon}{2N}\sum_{i,j=1}^{N}[1 - \cos(\theta_i - \theta_j)]$$



Isolated system – temperature is conserved



System in contact with thermal bath

Summary

1. Long-range interactions are important in many physical systems, both in nature and in man-made apparatus.
2. They have many remarkable properties which are poorly studied (QQS, negative specific heat, etc.)
3. The long-range interacting systems can be studied in the frames of the two approaches, the HMF model and the Lynden-Bell theory.
4. The most systems in nature are not isolated and should be considered in the canonical ensembles.
5. An interesting case of non-isolated system with QSS is Free-Electron Laser. I will focus on studying its properties in detail.
6. Both approaches, the HMF and the Lynden-Bell, will be used.
7. As a first step, the HMF system under the action of an external magnetic field is under study.

Thank you for attention!